Available online at www.sciencedirect.com
SCIENCE DIRECT®
European Economic Review 50 (2006) 909–936
EUROPEAN ECONOMIC REVIEW
www.elsevier.com/locate/eer

Evaluating the performance of the search and matching model

Eran Yashiv*

Tel Aviv University, Israel and CEPR

Received 29 September 2005; accepted 6 March 2006

Abstract

Does the search and matching model fit aggregate U.S. labor market data? While the model has become an important tool of macroeconomic analysis, recent literature pointed to some significant failures in accounting for the data. This paper aims to answer two questions: (i) Does the model fit the data, and, if so, on what dimensions? (ii) Does the data "fit" the model, i.e. what are the data which are relevant to be explained by the model?

The analysis shows that the model fits certain specifications of the data on many dimensions, though not on all. This includes capturing the high persistence and high volatility of most of the key variables, the negative co-variation of unemployment and vacancies, and the behavior of the worker job finding rate. A key role in this fit is played by the convexity of hiring costs and the stochastic properties of the separation rate. The latter is a major component of the rate discounting the future value of the job-worker match.

The paper offers a workable, empirically grounded version of the model for the analysis of aggregate U.S. labor market dynamics.

JEL classification: E24; E32; J32; J63

Keywords: Search; Matching; U.S. labor market; Vacancies; Labor market flows; Business cycles

1. Introduction

The importance of the role played by frictions in labor market dynamics and in macroeconomic fluctuations is increasingly recognized. The key modelling tool in this

*Tel.: +972 3 6409233.
E-mail addresses: yashiv@post.tau.ac.il, e.yashiv@lse.ac.uk.

0014-2921/$ - see front matter
doi:10.1016/j.euroecorev.2006.03.001

context is the aggregate search and matching model developed by Diamond, Mortensen and Pissarides (see Mortensen and Pissarides, 1999 and Yashiv, 2006a for surveys and Pissarides, 2000 for a detailed exposition). Recent papers, however, have questioned the model's empirical performance in U.S. data. This literature poses doubts with respect to the model's ability to account for the observed behavior of key variables—unemployment, vacancies, worker flows, wages, and the duration of unemployment. The aims of this paper are to answer two questions: (i) Does the model fit the data, and, if so, on what dimensions? (ii) Does the data "fit" the model, i.e. which data are relevant in the context of the model? The basic motivation is to try to understand whether the model—beyond its theoretical appeal—is indeed useful for the analysis of the U.S. labor market. The idea is to make precise the dimensions on which the model does well and those on which it does not do well or even fails. The analysis produces a workable, empirically grounded version of the model that may be used to analyze U.S. data and study policy questions.

The paper uses a partial equilibrium model and a reduced-form VAR of the actual data to specify the driving shocks. This 'agnostic' approach precludes the possibility that labor market dynamics will be affected by misspecifications in other parts of a more general macroeconomic model. Thus it does not take a particular stand on the sources of the driving shock processes nor does it formulate an explicit structure for the rest of the macroeconomy. For the labor market it uses a structural approach, specifying agents' objectives and constraints, their optimal behavior, and the dynamic paths of key variables in equilibrium. The structural parameters quantify the degree of frictions. The data question mentioned above is treated by looking at alternative formulations of the pool of searching workers, taking into account also non-employed workers outside the official unemployment pool. The paper uses newly available gross worker flows data that are compatible with the model's formulations, rather than widely used vacancy data, which are shown to be inconsistent with the concepts of the model. The model's implied second moments are compared to U.S. data in terms of persistence, co-movement, and volatility.

I find that for the most part the model fits U.S. labor market data relatively well. This includes capturing the high persistence and high volatility of most of the key variables, the negative co-variation of unemployment and vacancies (the 'Beveridge curve'), and the behavior of the worker job finding rate. A key role in this fit is played by the convexity of hiring costs and the stochastic properties of the separation rate. The latter is a major component of the rate discounting the future value of the job-worker match, and it is this discounting role that makes it so important.

The paper makes the following contributions: First, it offers an empirically grounded model of aggregate U.S. labor market dynamics as implied by the search and matching model. Doing so it provides macroeconomists some guidance concerning the relevant "building block" for modelling the labor market. Second, by calibrating the model and evaluating it against alternative formulations of the data, it is able to show on what dimensions the model fits the data and which data series are the relevant ones. Third, particular specifications of the model are able to replicate key empirical regularities in U.S. data, which other models have been unable to capture; the reasons for this improved performance are discussed. In terms of the calibration-simulation methodology, there is an innovation in the "agnostic" approach taken with respect to the formulation of shocks.

The paper proceeds as follows: Section 2 presents the search and matching model. At the end of this section, I point out the contributions of Dale Mortensen in the current, macroeconomic context. Section 3 discusses the dynamic formulation of the model. Section 4 discusses the data series, presents the properties of the data that are to be matched, and proposes alternative formulations of the data to be used. Section 5 calibrates the model and evaluates the model-data fit. Section 6 examines the mechanism underlying the results, deriving the modelling and data lessons for using the model to study the U.S. labor market, and comparing the results to recent literature. Section 7 concludes.

2. The search and matching model

In this section I briefly present a stochastic, discrete-time version of the prototypical[1] search and matching model.[2] Note that the standard model is chosen intentionally, as the idea is to evaluate its fit with the data. Examination of alternative modelling routes is left for future research.

2.1. The basic set-up

There are two types of agents: unemployed workers (U) searching for jobs and firms recruiting workers through vacancy creation (V). Firms maximize their intertemporal profit functions with the choice variable being the number of vacancies to open. Each firm produces a flow of output (F), paying workers wages (W) and incurring hiring costs (Γ). Workers and firms are faced with different frictions such as different locations leading to regional mismatch or lags and asymmetries in the transmission of information. These frictions are embedded in the concept of a matching function which produces hires (M) out of vacancies and unemployment, leaving certain jobs unfilled and certain workers unemployed. Workers are assumed to be separated from jobs at a stochastic, exogenous rate, to be denoted by δ. The labor force (L) is growing with new workers flowing into the unemployment pool. The set-up, whereby search is costly and matching is time-consuming, essentially describes the market as one with trade frictions. Supply and demand are not equilibrated instantaneously, so at each date t there are stocks of unemployed workers and vacant jobs. The model assumes a market populated by many identical workers and firms. Hence I shall continue the discussion in terms of "representative agents." Each agent is small enough so that the behavior of other agents is taken as given.

2.2. Matching

A matching function captures the frictions in the matching process; it satisfies the following properties:

$$M_{t,t+1} = \widetilde{M}(U_t, V_t),$$
$$\frac{\partial \widetilde{M}}{\partial U} > 0, \quad \frac{\partial \widetilde{M}}{\partial V} > 0. \tag{1}$$

[1] One important addition to the standard analysis is a convex formulation for the hiring costs function.

[2] A detailed exposition may be found in Pissarides (2000). The stochastic, discrete time formulation presented here follows Yashiv (2004).

Empirical work (see the survey by Petrongolo and Pissarides, 2001) has shown that a Cobb–Douglas function is useful for parameterizing it

$$M_{t,t+1} = \mu U_t^{\sigma} V_t^{1-\sigma}, \tag{2}$$

where μ stands for matching technology. The parameter σ reflects the relative contribution of unemployment to the matching process and determines the elasticity of the hazard rates with respect to market tightness V_t/U_t.[3]

2.3. Firms

Firms maximize the expected, present value of profits (where all other factors of production have been "maximized out"):

$$\max_{\{V\}} \mathrm{E}_0 \sum_{t=0}^{\infty} \left(\prod_{j=0}^{t} \beta_j \right) [F_t - W_t N_t - \Gamma_t], \tag{3}$$

where $\beta_j = 1/(1 + r_{t-1,t})$.

This maximization is done subject to the employment dynamics equation given by

$$N_{t+1} = (1 - \delta_{t,t+1}) N_t + Q_{t,t+1} V_t. \tag{4}$$

The main F.O.C are[4]

$$\frac{\partial \Gamma_t}{\partial V_t} = Q_{t,t+1} \mathrm{E}_t \Lambda_t, \tag{5}$$

$$\Lambda_t = \mathrm{E}_t \beta_{t+1} \left[\frac{\partial F_{t+1}}{\partial N_{t+1}} - \frac{\partial \Gamma_{t+1}}{\partial N_{t+1}} - W_{t+1} - N_{t+1} \frac{\partial W_{t+1}}{\partial N_{t+1}} \right] + \mathrm{E}_t (1 - \delta_{t+1,t+2}) \beta_{t+1} \Lambda_{t+1}. \tag{6}$$

The first, intratemporal condition (Eq. (5)) sets the marginal cost of hiring $\partial \Gamma_t / \partial V_t$ equal to the expected value of the multiplier times the probability of filling the vacancy. The second, intertemporal condition (Eq. (6)) sets the multiplier equal to the sum of the expected, discounted marginal profit in the next period

$$\mathrm{E}_t \beta_{t+1} \left[\frac{\partial F_{t+1}}{\partial N_{t+1}} - W_{t+1} - \frac{\partial \Gamma_{t+1}}{\partial N_{t+1}} - N_{t+1} \frac{\partial W_{t+1}}{\partial N_{t+1}} \right]$$

and the expected, discounted (using also δ) value of the multiplier in the next period $\mathrm{E}_t (1 - \delta_{t+1,t+2}) \beta_{t+1} \Lambda_{t+1}$.[5]

---

[3]The hazard rates—P, the worker probability of finding a job, and Q, the firm's probability of filling the vacancy—are derived as follows:

$$P_{t,t+1} = \frac{M_{t,t+1}}{U_t} = \mu \left(\frac{V_t}{U_t} \right)^{1-\sigma},$$

$$Q_{t,t+1} = \frac{M_{t,t+1}}{V_t} = \mu \left(\frac{V_t}{U_t} \right)^{-\sigma}.$$

[4]Other F.O.C are the flow constraint (4) and a transversality condition.

[5]Note that because I postulate that Γ depends on N (see below), the net marginal product for the firm depends on N. This marginal product is part of the match surplus bargained over, and therefore part of the wage solution discussed below. Hence the term $\partial W_{t+1}/\partial N_{t+1}$, usually absent, is not zero in this formulation.

For production I assume a standard Cobb–Douglas function

$$F_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{7}$$

where A is technology and K is capital.

Hiring costs refer to the costs incurred in all stages of recruiting: the cost of posting, advertising and screening—pertaining to all vacancies (V), and the cost of training and disrupting production—pertaining to actual hires (QV). For the functional form I use a power function formulation. This modelling relates to the same rationale being used in the capital adjustment costs/Tobin's Q literature. It emerged as the preferred one—for example as performing better than polynomials of various degrees—in structural estimation of this model reported in Yashiv (2000a,b) and in Merz and Yashiv (2006). The former studies used an Israeli data-set that is uniquely suited for such estimation with a directly measured vacancy series that fits well the model's definitions. The latter study used U.S. data. Formally this function is given by

$$\Gamma_t = \frac{\Theta}{1+\gamma}\left(\frac{\phi V_t + (1-\phi)Q_t V_t}{N_t}\right)^{\gamma+1} F_t. \tag{8}$$

Hiring costs are a function of the weighted average of the number of vacancies and the number of hires. They are internal to production and hence are proportional to output. Note that Θ is a scale parameter, ϕ is the weight given to vacancies as distinct from actual hires, and γ expresses the degree of convexity.[6]

The function is linearly homogenous in V, N and F. It encompasses the cases of a fixed cost per vacancy (i.e. linear costs, $\gamma = 0$) and increasing costs ($\gamma > 0$). Note, in particular, two special cases: when $\gamma = 0$ and $\phi = 1$, I get $\Gamma_t = \Theta V_t (F_t/N_t)$, which is the standard specification in much of the literature (for example, in Pissarides, 2000). When $\gamma = 1$, I get the quadratic formulation $\Gamma_t = \frac{\Theta}{2}((\phi V_t + (1-\phi)Q_t V_t)/N_t)^2 F_t$, which is analogous to the standard formulation in "Tobin's q" models of investment where costs are quadratic in I/K. In Yashiv (2006b) I further investigate the importance of using $\gamma > 0$ relative to $\gamma = 0$.

2.4. Wages

In this model, the matching of a worker and a vacancy against the backdrop of search costs, creates a joint surplus relative to the alternatives of continued search. Following Diamond (1982), the prototypical search and matching model derives the wage (W) as the Nash solution of the bargaining problem of dividing this surplus between the firm and the worker (see the discussion in Pissarides, 2000, Chapters 1 and 3). Note that I follow the prototypical model in the way it models wage setting in general, and in the specific form of wage bargaining in particular. While alternative mechanisms have been suggested, it is beyond the scope of this paper to examine them.

[6] Its derivatives, used below, are given by

$$\frac{\partial \Gamma_t}{\partial V_t} = \Theta(\phi + (1-\phi)Q_t)\left(\frac{\phi V_t + (1-\phi)Q_t V_t}{N_t}\right)^{\gamma} \frac{F_t}{N_t},$$

$$\frac{\partial \Gamma_t}{\partial N_t} = \Theta\left(\frac{\phi V_t + (1-\phi)Q_t V_t}{N_t}\right)^{\gamma+1} \frac{F_t}{N_t}\left[\frac{1-\alpha}{1+\gamma} - 1\right].$$

Formally this wage is

$$W_t = \operatorname{argmax}(J_t^N - J_t^U)^{\xi}(J_t^F - J_t^V)^{1-\xi}, \tag{9}$$

where J^N and J^U are the present value for the worker of employment and unemployment respectively; J^F and J^V are the firm's present value of profits from a filled job and from a vacancy respectively; and $0<\xi<1$ reflects the degree of asymmetry in bargaining.

Using the approach of Cahuc et al. (2004) to solve (9) taking into account the fact that $\partial W_{t+1}/\partial N_{t+1} \neq 0$, the wage is given by[7]

$$W_t = \xi \left(\begin{array}{c} (1-\alpha)A\left(\frac{K_t}{N_t}\right)^{\alpha}\left[\frac{1}{1-\alpha\xi} + \Theta\left(\frac{\phi V_t + (1-\phi)Q_t V_t}{N_t}\right)^{\gamma+1} \frac{\alpha+\gamma}{(1+\gamma)(1-\alpha)(1-\xi(1+\alpha+\gamma))}\right] \\ +P_{t,t+1}\Lambda_t \end{array} \right) + (1-\xi)b_t \tag{10}$$

where b is the income of the unemployed, such as unemployment benefits.

I assume that b_t is proportional to W_t i.e. $b_t = \tau W_t$, so τ may be labelled the "replacement ratio." Denoting $\xi/(1-(1-\xi)\tau)$ by η and formulating the wage in terms of the labor share in income (by dividing this wage by the average product) I get

$$W_t = s_t \frac{F_t}{N_t},$$
$$s_t = \eta \left(\begin{array}{c} (1-\alpha)\left[\frac{1}{1-\alpha\xi} + \Theta\left(\frac{\phi V_t + (1-\phi)Q_t V_t}{N_t}\right)^{\gamma+1} \frac{\alpha+\gamma}{(1+\gamma)(1-\alpha)(1-\xi(1+\alpha+\gamma))}\right] \\ +P_{t,t+1}\lambda_t \end{array} \right), \tag{11}$$

where $\lambda_t = \Lambda_t/(F_t/N_t)$.

This wage solution has the following properties:

a. Wages are a complicated function of productivity F_t/N_t, with the latter appearing directly and via the s_t term.
b. Wages and the wage share increase with worker bargaining power (ξ) or replacement ratio (τ) as expressed by η.
c. Wages and the wage share are a positive function of the match's future value given by $P_{t,t+1}\lambda_t$. Thus wages are positively related to the asset value of the match.

2.5. Equilibrium

The stocks of unemployment and employment and the flow of hiring emerge as equilibrium solutions. Solving the firms' maximization problem yields a dynamic path for vacancies; these and the stock of unemployment serve as inputs to the matching function; matches together with separation rates and labor force growth change the stocks of employment and unemployment.

This dynamic system may be solved for the five endogenous variables V, U, M, N and W given initial values U_0, N_0 and given the path of the exogenous variables. As noted above,

[7]The solution entails postulating the asset values of a filled job and of a vacant job for the firm and the asset values of employment and unemployment for the worker in (9). See the technical appendix at http://www.tau.ac.il/~yashiv/research.html for details of the derivation.

this is a partial equilibrium model. The exogenous variables include the worker's marginal product, the discount factor, the labor force, and the separation rate. If the production function is CRS and if the capital market is perfect—as I shall assume—the capital–labor ratio will be determined in equilibrium at the point where the marginal product of capital equals the interest rate plus the rate of depreciation. This in turn will determine production and the marginal product of labor. This set-up is consistent with several different macroeconomic models. For example Merz (1995) and Andolfatto (1996) have shown that a special case of this model may be combined with traditional elements of RBC models to yield a dynamic general equilibrium model. In these models the interest rate equals the marginal rate of intertemporal substitution in consumption.

In the following section I solve explicitly for a stochastic, dynamic equilibrium using a stochastic structure for the exogenous variables.

2.6. The contributions of Dale Mortensen

Dale Mortensen has made fundamental contributions to the model presented above. The first one, in the celebrated 1970 "Phelps volume" (Mortensen, 1970), introduced the flow approach to the labor market and incorporated search costs. In that paper, the firm's intertemporal choice was shown to be akin to investment with adjustment costs.

In a number of papers he analyzed the issues of search and surplus sharing: In Mortensen (1978) he presented a framework for analyzing the interrelationship between the choice of search strategies by the two parties involved in an existing match and the nature of the wage bargaining problem. The problem of searching for a preferred partner was formulated as two-person game played by the worker and the employer involved in the existing match. The paper compares the non-cooperative Nash solution of the game to the joint wealth maximizing search strategies. The main findings were that quit and dismissal rates are higher in the former relative to the latter, in the absence of a provision for compensation. These turnover rates depend on wages in the non-cooperative case, but are independent of wages in the joint wealth maximizing case. Mortensen (1982a, 1982b) dealt with the following issue: Two types of agents search for each other in order to exploit a joint production opportunity. The way the value of the match is divided affects the incentives of the agents when searching for the match. An equal division of the surplus was shown by Mortensen (1982a) to yield too little effort by both types of agents. Mortensen (1982b) showed that search effort made by all agents in a Nash equilibrium is efficient when the matchmaker receives all the surplus.

Finally, the model was expanded to incorporate idiosyncratic productivity and endogenous separations in Mortensen and Pissarides (1994).

3. The dynamic model

I use a log-linear approach, transforming the non-linear problem into a first-order, linear, difference equations system through approximation and then solving the system using standard methods. To abstract from population growth, in what follows I cast all labor market variables in terms of rates out of the labor force L_t, denoting them by lower case letters. Productivity growth is captured by the evolution of A, which enters the model through the dynamics of F_t/N_t, so I divide all variables by the latter. This leaves a system that is stationary and is affected by shocks to labor force *growth*, to productivity *growth*, as well as to the interest rate and to the job separation rate, to be formalized below.

The model has four exogenous variables. These are productivity growth ($G^X = (F_{t+1}/N_{t+1})/(F_t/N_t)$), labor force growth ($G^L = L_{t+1}/L_t$), the discount factor ($\beta$) and the separation rate (δ). These affect dynamics as follows: productivity growth G^X raises the match surplus; the discount rate β (related to the rate of interest) and the separation rate δ are components of the relevant discount rate used in computing the present value of the match. Empirical testing reveals that G^L can be modelled as white noise around a constant value. When I tried to add it as a stochastic variable to the framework below the results were not affected, so I treat it as a constant. It is the other three variables that inject shocks into this system. As mentioned, I do not formulate the underlying shocks structurally. Instead, I postulate that they follow a first-order VAR (for each variable Y, I use the notation $\widehat{Y}_t = (Y_t - Y)/Y \approx \ln Y_t - \ln Y$ where Y is the steady state value, so all variables are log deviations from steady state)

$$\begin{bmatrix} \widehat{G}^X_{t+1} \\ \widehat{\beta}_{t+1} \\ \widehat{\delta}_{t+1} \end{bmatrix} = \Pi \begin{bmatrix} \widehat{G}^X_t \\ \widehat{\beta}_t \\ \widehat{\delta}_t \end{bmatrix} + \Sigma. \tag{12}$$

In the empirical section below I use reduced-form VAR estimates of the data to quantify the coefficient matrix Π and the variance–covariance matrix of the disturbances Σ. Thus the current model is consistent with both RBC-style models that emphasize technology shocks as well as with models that emphasize other shocks.[8] Note also that the shocks may interact through the off-diagnonal elements in Π and Σ.

In the non-stochastic steady state the rate of vacancy creation is given by

$$\Theta(\phi + (1-\phi)Q)\left(\frac{\phi V + (1-\phi)QV}{N}\right)^{\gamma} = Q\,\frac{G^X\beta}{[1-(1-\delta)G^X\beta]}\pi. \tag{13}$$

The LHS are marginal costs; the RHS is the match asset value. It is the probability of filling a vacancy (Q) times the marginal profits accrued in the steady state. The latter are the product of per-period marginal profits π and a discount factor $G^X\beta/(1 - G^X\beta(1-\delta))$ that takes into account the real rate of interest, the rate of separation and productivity growth.

Labor market flows in the steady state are given by

$$(\delta + G^L - 1) = \frac{m}{n} = \frac{Qv}{n}. \tag{14}$$

This expression equates the rate of increase in employment through matching with the sum of the rates of separation and increase in the labor force. From this equation the rate of unemployment in equilibrium is given by

$$u = \frac{\delta + (G^L - 1)}{\delta + (G^L - 1) + P}. \tag{15}$$

[8] Note that $\widehat{G}^X_{t+1}$ is a stationary $I(0)$ variable. This is the maintained assumption in the literature using reduced-form VAR involving average labor productivity, such as Christiano et al. (2003).

I log-linearly approximate the deterministic version of the F.O.C in the neighborhood of this steady state. The resulting system is a first-order, linear, difference equation system in the state variable $\widehat{n}$ and the co-state $\widehat{\lambda}$ with three exogenous variables, $\widehat{G}^X$, $\widehat{\beta}$ and $\widehat{\delta}$. The matrices of coefficients are defined by the parameters $\alpha, \Theta, \gamma, \phi, \mu, \sigma$ and η and by the steady-state values of various variables. The solution of this system enables me to solve for the control variable—vacancies, and for other variables of interest, such as unemployment, hires, the matching rate, and the labor share of income.

4. U.S. labor market data

In relating the model to U.S. data, a number of important issues arise. The following discussion shows that the different variables have multiple representations in the data and some are not consistent with the concepts of the model. The idea is to select those series that do match these concepts and to examine alternative representations wherever relevant.

4.1. The relevant pool of unemployment

In order to see how the model relates to the data, a key issue that needs to be resolved is the size of the relevant pool of searching workers. The question is whether this pool is just the official unemployment pool or a bigger one. The model speaks of two states—employment and unemployment; in the model matches are flows from unemployment to employment and separations are flows from employment to unemployment.[9] In the actual data—taken from the CPS—several important issues arise:

(i) Flows between the pool out of the labor force and the labor force, including flows directly to and from employment, are sizeable. Recent data indicate that unemployment to employment flows are on average 1.9 million workers per month, while out of the labor force to employment flows are 1.5 million workers per month on average.
(ii) Clark and Summers (1979) have argued that there is substantial misclassification of unemployment status and that ‘‘many of those not in the labor force are in situation effectively equivalent to the unemployed’’ (p. 29), providing several measures to substantiate this claim. Data collectors were aware of this issue: following the recommendations of the Gordon committee, which recognized that there could be some form of ‘‘hidden unemployment,’’ beginning in January 1967 the CPS included questions on out of the labor force people who could potentially be defined as unemployed. This generated a quarterly series on people that responded affirmatively to the question if they ‘‘wanted a job now’’ (I report this series below).
(iii) Working on the re-designed CPS data in the period 1994–1998, Jones and Riddell (2000) further demonstrate the importance of these distinctions. Key results include estimates of the hazard rates for three worker groups: the unemployed, the marginally attached, and the unattached, the last two being officially classified as out of the labor force. Their monthly hazard rates are 0.20–0.35 for the unemployed, 0.10–0.20 for the marginally attached, and below 0.05 for the unattached. Various tests indicate that these are indeed three distinct states.

[9]Additionally, labor force growth (with new participants joining the unemployment pool) is an exogenous variable.

(iv) The out of the labor force flows exhibit markedly different cyclical properties relative to flows between employment and unemployment: The unemployment to employment flows are countercyclical while the out of the labor force to employment flows are procyclical.

Given this evidence, it appears natural to consider pools of workers outside the labor force when coming to study labor market dynamics and worker flows. The question is how to add the unobserved 'unemployment' pool from out of the labor force to the 'official' pool of unemployment. The strategy I use in the empirical work is as follows. I look at the three "natural" candidate series: (i) Official unemployment, (ii) official unemployment and the "want a job now" category and (iii) the entire working age population. As the relevant pool may lie between the second and third cases and as no measured pool is available, I look at two additional specifications that try to approximate that pool. Thus, starting from case (ii), I add 15% and 30% of the remaining workers from the out of the labor force pool.

Fig. 1 shows the resulting series, including the official unemployment rate, for the period in which the series exist. Table 1 provides sample statistics for these five series.

Two features stand out: While the mean of the series evidently rises with the expansion of the pool, the volatility hardly changes going from the official + want a job pool to the larger pools, and the series are highly correlated (though the correlation slightly declines as the pool expands).

In what follows, I look at the properties of the data under these different specifications. In Section 5 below, I compare the performance of the calibrated model against these alternative specifications and decide on a benchmark specification for the subsequent analysis.

4.2. The appropriate measure of job vacancies

The relevant concept of vacancies in the model is the one relating to those vacancies that are to be filled with workers from outside the employment pool (I shall denote it by V^{UN}).

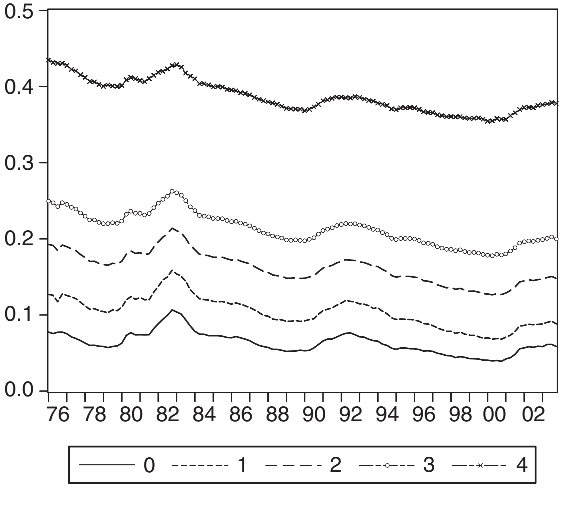

Fig. 1. Alternative specifications of the unemployment rate.

Table 1
Stochastic properties of alternative measures of the pool of searching workers (Quarterly data)

Pool of workers	Mean	Std.	Correlations				
			0	1	2	3	4
0 = official pool U^o	0.06	0.014	1				
1 = official + want a job now	0.10	0.020	0.976	1			
2 = official + want a job + $0.15(POP - N - U^0)$	0.16	0.021	0.959	0.992	1		
3 = official + want a job + $0.3(POP - N - U^0)$	0.21	0.021	0.937	0.977	0.996	1	
4 = $POP - N$	0.39	0.022	0.856	0.908	0.953	0.977	1

Notes: *POP* is working age population; *N* is civilian employment.

But the available and widely used data series pertains to another concept, which also includes vacancies that are subsequently filled with workers moving from job to job (to be denoted by V_t^{NN}).[10] Simply this can be expressed as follows:

$$V_t^{\text{tot}} = V_t^{UN} + V_t^{NN}.$$

The V_t^{tot} series in the U.S. economy has two representations: one is the index of Help Wanted advertising in newspapers published by the Conference Board. A newer series is the job openings series available from the BLS since December 2000 using the job openings and labor turnover survey (JOLTS). The two series have a correlation of 0.88 over 37 monthly observations. Fig. 2 plots the longer vacancy series as well as gross flows of workers from outside employment (unemployment and out of the labor force) to employment, which can be taken to represent $Q \times V^{UN}$. The latter was recently compiled at the Boston Fed based on CPS data (see Bleakley et al., 1999). The figure shows the two series normalized. Table 2 presents the coefficient of variation (or standard deviation) and autocorrelation for the two vacancy series and for the hiring series.

The hiring flows series is negatively correlated with the two vacancy series: −0.27 with the JOLTS series and −0.36 with the Help Wanted ads series. The flows series also appears to be much less persistent than the vacancies series: hiring is substantially less persistent in levels and even more so in HP-filtered terms; it has similar persistence in market tightness terms. Compared to the volatility of the Help Wanted Index, hiring volatility is about a seventh in levels, a fifth in HP-filtered terms, and about a third in ratios to unemployment terms. It is similarly less volatile than the JOLTS data series.

This comparison—between V^{tot} and $Q \times V^{UN}$—suggests that V^{tot} may either be very different from V^{UN} or that the behavior of Q generates these discrepancies. Without direct measures of V^{UN} it is not possible to determine which explanation holds true but there is indirect evidence. In Yashiv (2006c) I examine this issue, showing that the evidence is consistent with the interpretation that the behavior of the Help Wanted Index, which relates to the broader V^{tot}, is different from the behavior of the relevant series here, V^{UN}. The present model, being an aggregate, representative firm-type of model, does not deal

[10]This should not be taken to mean that firms post two types of vacancies. The idea is just to say that some vacancies are ex post filled by previously unemployed workers and the rest by previously employed workers moving directly from job to job.

Fig. 2. Vacancy index and hires (normalized).

Table 2
Stochastic properties of vacancies and hires (Quarterly data)

	Levels	Individual samples			Common sample	
		Sample coverage	$\frac{\text{std.}}{\text{mean}}$	Auto-correlation	$\frac{\text{std.}}{\text{mean}}$	Auto-correlation
V	Help wanted index	1951:I–2003:IV	0.34	0.98	0.27	0.56
V	JOLTS job openings	2001:I–2003:IV	0.24	0.61	0.24	0.61
QV	Hires	1976:II–2003:IV	0.05	0.61	0.03	0.17
	Logs, HP-filtered		std.	Auto-correlation		
V	Help wanted index	1951:I–2003:IV	0.14	0.89		
V	JOLTS job openings	2001:I–2003:IV	0.28	0.60		
QV	Hires	1976:II–2003:IV	0.03	0.05		
	Ratio to unemployment		$\frac{\text{std.}}{\text{mean}}$	Auto-correlation	std.	Auto-correlation
V/U	Help wanted index	1951:I–2003:IV	0.44	0.95	0.47	0.62
V/U	JOLTS job openings	2001:I–2003:IV	0.35	0.66	0.35	0.66
QV/U	Hires	1976:II–2003:IV	0.16	0.94	0.15	0.64

with job to job movements (as is also the case for the recent literature to be cited below), and thus the Help Wanted Index is not the appropriate series to use, as it probably does not behave like the relevant series (V^{UN}). Given that the latter is unobserved, I focus on the observed worker flow series (i.e., on $Q \times V^{UN}$) whenever comparing the model to the data. The above discussion also implies that care must be taken when using or discussing vacancy data in the U.S. economy.

4.3. The flow of hires and alternative representations of the job finding rate

The discussion on the relevant pool of unemployment makes it clear that it is important to analyze the flow of matches or hires using out of the labor force to employment flows as well as unemployment to employment flow. This implies that in addition to different unemployment pools U, there will be different matching flows M and consequently different worker job finding rates $P = M/U$. Fig. 3 presents these different rates. Table 3 compares them to the estimates of the afore-cited Jones and Riddell (2000) micro-based, CPS 1994–1998 study.

The 0 and 1 series represent the ones derived from the official rate and from the official+want a job series respectively. They imply unemployment durations of 17.4 weeks and 16.5 weeks, respectively. In comparison, official BLS data on duration indicate 15.1 weeks on average (with 2.5 weeks standard deviation) in the same period. The other series—specifications 2, 3 and 4—evidently imply higher durations (28.0, 39.0 and 91.8 weeks, respectively) and lower job finding rates. All series are highly correlated with the notable exception of specification 4, which is almost uncorrelated with the 0 and 1 specifications and weakly to moderately correlated with the 2 and 3 specifications. The official pool has a slightly higher job finding rate than the upper bound of the Jones and Riddel estimate (i.e., 0.80 compared to 0.73). From the three intermediate pools, specification 2 has a rate (0.47) that is consistent with a mixture of the unemployed and the marginally attached. The largest pool, that includes all non-employed workers (specification 4 here), has a rate of 0.14 that seems to be consistent with a mixture of the unattached with the other two groups.

4.4. Wages

The existence of diverse data series for wages with different cyclical properties was noted by several papers, for example, Abraham et al. (1999) and Krueger (1999). The discussion

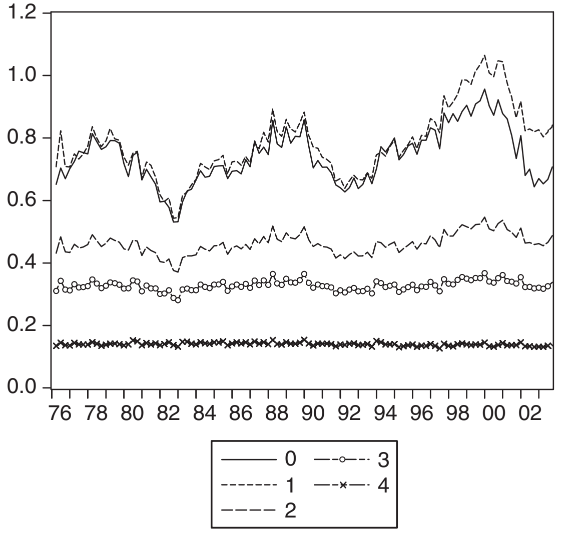

Fig. 3. Job finding rate.

Table 3
Worker job finding rates P (Quarterly 1994–1998)

Specification	Average P	Jones and Riddell (2000) specification	Range of estimated hazard rates
0	0.80	Unemployed	0.49–0.73
1	0.82		
2	0.47	Marginally attached	0.27–0.49
3	0.39		
4	0.14	Unattached	Around 0.10, always below 0.14

Notes: The five specifications, 0 to 4, correspond to the definitions in Table 1.

in these papers does not lead to any definite conclusion as to which series is the most appropriate. Fig. 4 illustrates one aspect of this issue by plotting BEA series of the labor share $s = W/(F/N)$, once using total compensation[11] and once using wages.

The series are correlated 0.83 but have a number of important differences: the wage series declines more over time, is lower by ten percentage points on average, and displays much more variation (coefficient of variation of 0.037 relative to 0.016 for the other series). It should also be noted that both series have very weak correlation with the cycle: the compensation series has -0.05 correlation with the employment rate and the wage series has a 0.12 correlation.

In what follows I use the compensation series as it takes all firm's wage-related costs, which is the relevant concept in the model.

4.5. Other data series

Fig. 5 shows the other data series to be used in the empirical work below: Productivity growth G^X (gross rate), the discount factor β, and the separation rate δ (separately for the official unemployment specification δ_0 and for the others δ).

For productivity growth I take the rate of change of GDP per worker; for the discount factor I take $\beta = 1/(1+r)$ where r is the cost of firm finance (weighted average of equity finance and debt finance); for the separation rate I take the flow out of employment divided by the stock of employment.

4.6. Data properties

Table 4 reports the first two moments of all relevant data series, including measures of persistence and co-movement. When looking at these moments it is important recall that the business cycle is most clearly manifested in the labor market—there is high correlation between employment and output and their volatility is similar. The table describes the key moments across the different specifications of the unemployment pool.

[11] Defined as total compensation of employees relative to GDP; this includes, beyond wages and salaries, supplements such as employer contribution for employee pension and insurance funds and employer contribution for government social insurance.

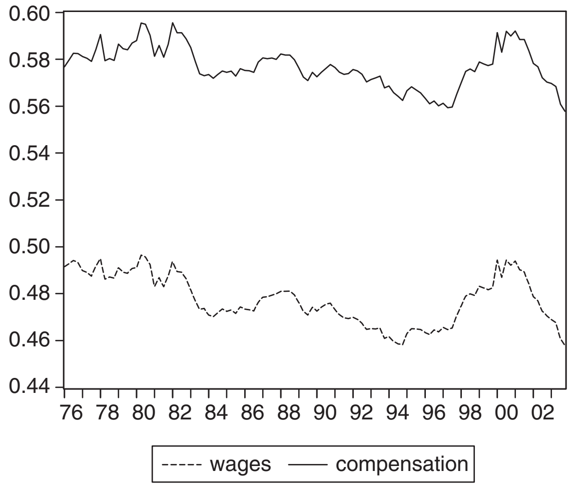

Fig. 4. The wage share s.

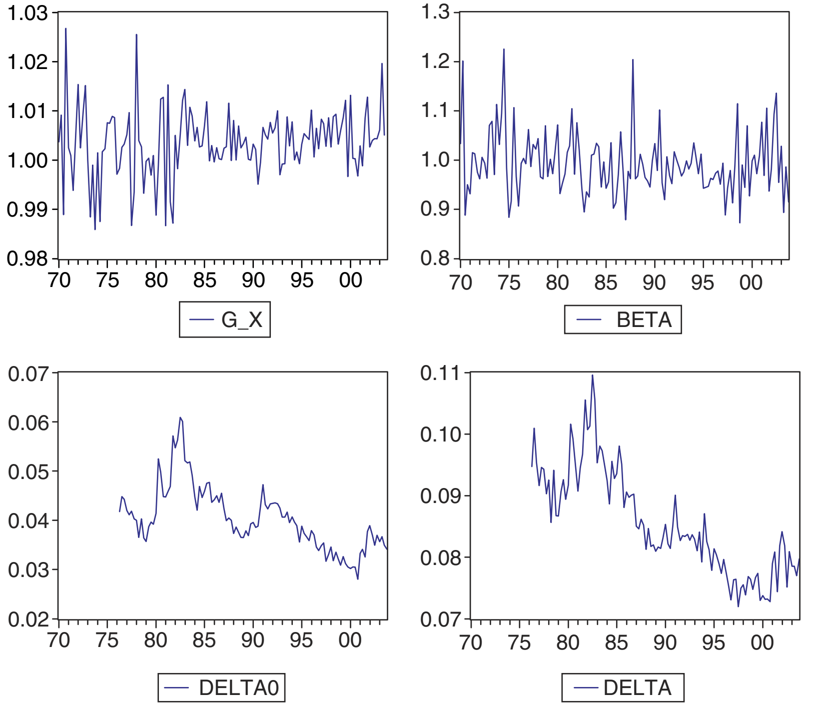

Fig. 5. $G^X, \beta, \delta_0, \delta$.

The following major properties can be said to characterize the data:

Persistence: All the main labor market variables are persistent: the rate of unemployment (and thus employment), hiring, separation, and the wage share all exhibit high persistence. At the same time productivity growth and the discount factor are not

Table 4
Data properties

a. Sample Means

	(0)	(1)	(2)	(3)	(4)
unemployment rate u	0.06	0.10	0.16	0.22	0.39
hiring rate rate $m = qv$	0.046	0.079	0.074	0.070	0.054
labor force growth $G^L - 1$	0.0043	0.0042	0.0041	0.0040	0.0036
separation rate δ	0.040	0.086			
productivity growth $G^X - 1$	0.0036				
discount factor β	0.993				
labor share s	0.58				

b. Persistence (auto-correlation)

	(0)	(1)	(2)	(3)	(4)
$\rho(\widehat{u}_t, \widehat{u}_{t-1})$	0.96	0.97	0.98	0.985	0.989
$\rho(\widehat{m}_t, \widehat{m}_{t-1})$	0.91	0.85	0.84	0.84	0.81
$\rho(\widehat{s}_t, \widehat{s}_{t-1})$	0.88				
$\rho(\widehat{G}^X_t, \widehat{G}^X_{t-1})$	−0.014				
$\rho(\widehat{\delta}_t, \widehat{\delta}_{t-1})$	0.92	0.89			
$\rho(\widehat{\beta}_t, \widehat{\beta}_{t-1})$	0.02				

c. Volatility (standard deviation)

	(0)	(1)	(2)	(3)	(4)
$\widehat{n}$	0.015	0.022	0.024	0.028	0.042
$\widehat{u}$	0.22	0.19	0.12	0.10	0.07
$\widehat{m}$	0.13	0.09	0.08	0.08	0.07
$\widehat{\frac{m}{u}}$	0.12	0.14	0.07	0.05	0.04
$\widehat{s}$	0.016				
$\widehat{G}^X$	0.007				
$\widehat{\delta}$	0.16	0.10			
$\widehat{\beta}$	0.06				

d. Co-movement (correlation)

	(0)	(1)	(2)	(3)	(4)
$\rho(\widehat{u}_t,\widehat{m}_t)$	0.92	0.81	0.86	0.88	0.87
$\rho(\widehat{u}_t,\widehat{P}_t)$	−0.91	−0.93	−0.80	−0.59	0.24
$\rho(\widehat{n}_t,\widehat{s}_t)$	−0.06	−0.16	−0.30	−0.39	−0.52
$\rho(\widehat{m}_t,\widehat{s}_t)$	0.27	0.45	0.45	0.44	0.41
$\rho(\widehat{n}_t,\widehat{G}^X_t)$	−0.10	−0.03	0.001	0.02	0.06
$\rho(\widehat{n}_t,\widehat{\beta}_t)$	0.15	0.10	0.04	0.00	−0.08
$\rho(\widehat{n}_t,\widehat{\delta}_t)$	−0.93	−0.88	−0.91	−0.92	−0.91

Notes: 1. All data are quarterly for the period 1970:I–2003:IV, except for hires and separations which begin in 1976:I and end in 2003:III. 2. The five specifications, columns (0)–(4), correspond to the definitions in Table 1.

persistent at all. Note that one of the three driving shocks—the separation rate—is persistent.

Volatility: (i) With only the officially unemployed considered searching (column 0), the volatility of the unemployment rate is the highest at 0.22 in terms of log levels; less volatile are the hiring and separation rates, which have comparable volatility at about 0.13–0.16; the discount factor has a volatility of 0.06 in the same terms; the volatility of the rate of employment ($n = N/L$) is of the same order of magnitude as that of the wage share at around 0.015 (in the same log terms); productivity growth at 0.007 has even lower volatility. Note that this is somewhat akin to investment behavior: The capital stock (here the employment stock) is much less volatile than investment (here the hiring flow). Note, too, the relatively high volatility of the rate of separation and of the discount factor, which are driving factors.

(ii) When moving across columns to a broader specification of the pool of searching workers some patterns change: In relative terms, hiring and separation become more volatile (at 0.07–0.10 in column 4) than unemployment (at 0.07). In absolute terms the employment rate becomes more volatile while unemployment, hiring and separation (in rates) all become less volatile.

Co-movement: (i) Hiring rates (m) co-vary positively with the unemployment rate, while the worker job finding rate (P) usually co-varies negatively with the same variable. This means that hiring flows, in rates, are counter-cyclical[12] but job-finding rates $P = m/u$ are pro-cyclical. This is so as in recessions U rises and P falls; $m = M/L = P \cdot U/(N + U)$ rises, as the effect of a rising U dominates the fall in P and any change in L.

(ii) Separation rates (δ) are counter-cyclical. Note that hiring and separation rates move together, unlike the widely known negative correlation between job creation and job destruction in the manufacturing sector.

(iii) The labor share in income (s) varies between a-cyclicality and counter-cyclicality (with respect to employment) according to the specification of the labor force. Throughout it covaries positively with the hiring rate. This means that when F/N rises in booms, the wage (W) rises by less and thereby the labor share (s) declines.

(iv) There is low correlation between employment and productivity growth, a fact which has received considerable attention in the RBC literature.

(v) Employment has low co-variation with the discount factor.

There are some non-intuitive aspects to these data moments: in booms hiring and separation rates fall and the labor share either does not change or falls. Hiring is strong at the same time as the share of wages is high.

5. Calibration and model-data fit

In this section I calibrate the model and examine its performance, taking into account the alternative formulations of the pool of searching workers discussed above. I begin by discussing calibration values in Section 5.1. I then (5.2) examine the performance of the model.

5.1. Calibration

There are three structural parameters that are at the focal point of the model and that reflect the operation of frictions. These are the matching function parameter

[12] This is also true for logged and HP-filtered variables.

σ (elasticity of unemployment), the wage parameter η, and the hiring function convexity parameter γ

For σ I use Blanchard and Diamond's (1989) estimate of 0.4. Structural estimation of the model using U.S. corporate sector data in Merz and Yashiv (2006) indicates a value of γ, the convexity parameter of the hiring cost function, around 2, i.e. a cubic function ($\gamma + 1 = 3$) for hiring costs. These costs fall on vacancies and on actual hires, with ϕ being the weight on the former. I follow the estimates in Yashiv (2000a) and set it at 0.3.

The wage parameter η depends on the asymmetry of the bargaining solution (ξ) and on the replacement ratio (τ). This is obviously a difficult case for calibration as ξ is not directly observed and τ depends not only on the value of benefits relative to wages but also on actual take-up rates. Following Anderson and Meyer (1997) I postulate a value of 0.25 for the latter; to test for robustness I tried a far higher value, finding that this change has a very small effect on the resulting moments. For ξ, rather than imposing it, I solve it out of the steady state relations.

For the production function labor parameter α I use a fairly traditional value of 0.68, which is also the structural estimate of this parameter in Merz and Yashiv (2006).

For the values of the exogenous variables I use sample average values. For the steady-state values of the endogenous variables, given the discussion above on the relevant unemployment pool, I modify n and u according to the specifications used above. Calibration of Q, the matching rate for vacancies, is problematic as there are no wide or accurate measures of vacancy durations for the U.S. economy. Using a 1982 survey, Burdett and Cunningham (1998) estimated hazard functions for vacancies both parametrically and semi-parametrically finding that the general form of the hazard function within the quarter is non-mononotic; based on their estimates the quarterly hazard rate should be in the range of 0.8–1. I thus take $Q = 0.9$ which is also the value used by Merz (1995) and Andolfatto (1996). This implies a particular steady state value for the vacancy rate (v). I use the average of the labor share in income s which is 0.58.

With the above values, I solve the steady state relations for the steady state vacancy rate v, the hiring cost scale parameter Θ, the matching function scale parameter μ, and the wage parameter ξ. I can then solve for the steady state values of market tightness v/u, the worker hazard rate P, per period profits π and the match asset value λ.

Table 5 summarizes the calibrated values for the different specifications of the unemployment pool.

Note two features of the implied results:

(i) The implied wage parameter η, encompassing the worker bargaining strength and the replacement ratio, varies between 0.4 and 0.6 across specifications.
(ii) Across specifications 1–4, per period profits (π) are around 0.07–0.13 (in average output F/N terms) and the asset value of the match (λ) is around 0.7–1.5. Given that $s = 0.58$ this means that asset values are about 1.2–2.6 the labor share in income. In other words the match is worth around 1–2.5 quarters of wages in present value terms.

As to the stochastic shocks, in order to get numerical values for the coefficient matrix Π and for the variance-co-variance matrix of Σ, I estimate a first-order VAR in labor productivity growth, the discount factor, and the rate of match separation. I estimate this VAR with the relevant data discussed above (see the appendix for definitions and sources).

Table 5
Baseline calibration values

a. Parameters, Exogenous Variables, and Steady State Values

Parameter/Variable	symbol	(0)	(1)	(2)	(3)	(4)
Production	$1-\alpha$	0.68				
Matching	σ	0.4				
Hiring (convexity)	γ	2				
Hiring (vacancy weight)	ϕ	0.3				
productivity growth	G^X-1	0.003536				
labor force growth	G^L-1	0.004296	0.004199	0.004078	0.003974	0.003631
discount factor	β	0.9929				
separation rate	δ	0.0404	0.0854			
Unemployment	u	0.063	0.104	0.164	0.217	0.395
Labor share	s	0.579				
Vacancy matching rate	Q	0.9				

b. Implied Values

		(0)	(1)	(2)	(3)	(4)
Matching scale parameter	μ	0.80	0.85	0.69	0.60	0.42
Hiring scale parameter	Θ	465	82	109	127	171
Wage bargaining parameter	ξ	0.37	0.41	0.45	0.49	0.56
Wage parameter	η	0.44	0.48	0.53	0.56	0.63
Vacancy rate	v	0.047	0.089	0.083	0.078	0.060
Market tightness	$\frac{v}{u}$	0.74	0.86	0.51	0.36	0.15
Workers' hazard	P	0.67	0.77	0.46	0.32	0.14
Profits	π	0.05	0.07	0.09	0.10	0.13
Asset Value	λ	1.02	0.73	0.96	1.12	1.49

Notes: 1. The implied values of v, μ, Θ and η are solved for using the steady state relationships. 2. The five specifications, columns (0) to (4), correspond to the definitions in Table 1.

5.2. Model-data fit

I now turn to examine the performance of the model.[13] Table 6 shows the moments implied by the model and those of the data (repeating the moments reported in Table 4). The following conclusions can be drawn:

Persistence: The model captures the fact that across all specifications u, m and s are highly persistent. The model tends to somewhat overstate this persistence.

Volatility: Column 1 captures very well the volatility of employment and unemployment. Hiring volatility is understated by the model; best performing is column 0 which captures three quarters of this volatility. As to the labor share, the model overstates its volatility with column (4) being the closest to the data. No single specification produces a high model-data fit for all variables.

[13] I use a modified version of a program by Craig Burnside in Gauss to solve the model (see Burnside, 1997).

Table 6
Model evaluation: Alternative specifications

		(0)	(1)	(2)	(3)	(4)
a. Model vs. data						
$\rho(\widehat{u}_t, \widehat{u}_{t-1})$	Data	0.962	0.971	0.980	0.985	0.989
	Model	0.989	0.983	0.990	0.993	0.996
$\rho(\widehat{m}_t, \widehat{m}_{t-1})$	Data	0.907	0.853	0.844	0.836	0.805
	Model	0.991	0.986	0.991	0.993	0.992
$\rho(\widehat{s}_t, \widehat{s}_{t-1})$	Data	0.884	0.884	0.884	0.884	0.884
	Model	0.982	0.976	0.988	0.992	0.996
$std(\widehat{n}_t)$	Data	0.015	0.022	0.024	0.028	0.042
	Model	0.020	0.021	0.032	0.042	0.072
$std(\widehat{u}_t)$	Data	0.218	0.188	0.124	0.100	0.069
	Model	0.298	0.182	0.165	0.151	0.110
$std(\widehat{m}_t)$	Data	0.125	0.085	0.083	0.080	0.073
	Model	0.097	0.051	0.035	0.022	0.018
$std(\widehat{s}_t)$	Data	0.016	0.016	0.016	0.016	0.016
	Model	0.089	0.056	0.052	0.049	0.038
$\rho(\widehat{u}_t, \widehat{m}_t)$	Data	0.920	0.810	0.860	0.880	0.870
	Model	0.997	0.997	0.998	0.999	−0.982
$\rho(\widehat{u}_t, \widehat{P}_t)$	Data	−0.909	−0.933	−0.802	−0.591	0.242
	Model	−0.999	−1.000	−1.000	−1.000	−1.000
$\rho(\widehat{n}_t, \widehat{s}_t)$	Data	−0.060	−0.160	−0.300	−0.390	−0.520
	Model	0.995	0.997	0.999	1.000	1.000
$\rho(\widehat{m}_t, \widehat{s}_t)$	Data	0.272	0.451	0.445	0.440	0.414
	Model	−0.985	−0.989	−0.996	−0.999	0.984
b. Model predictions						
$\rho(\widehat{v}_t, \widehat{v}_{t-1})$		0.952	0.961	0.986	0.992	0.996
$\rho(\widehat{q}_t, \widehat{q}_{t-1})$		0.988	0.981	0.989	0.992	0.996
$std(\widehat{v}_t)$		0.040	0.037	0.052	0.064	0.103
$std(\widehat{q}_t)$		0.135	0.088	0.087	0.086	0.085
$\rho(\widehat{u}_t, \widehat{v}_t)$		−0.956	−0.985	−0.998	−1.000	−0.999
$\rho(\widehat{v}_t, \widehat{q}_t)$		−0.966	−0.990	−0.999	−1.000	−1.000
$\rho(\widehat{v}_t, \widehat{s}_t)$		0.981	0.995	0.999	1.000	0.999

Notes: The five specifications, columns (0) to (4), correspond to the definitions in Table 1.

Co-movement: Under most specifications of the model, the counter-cyclical behavior of hiring and the pro-cyclical behavior of the worker job finding rate are well captured. But the behavior of the labor share is not captured: while in the data it is a-cyclical to counter-cyclical (across specifications of the unemployment pool) and co-varies moderately with the hiring rate, it is strongly pro-cyclical in the model and has a strong negative relationship with hiring, expect for column 4 where it is positive but overstated.

Overall fit: The model captures the persistence, volatility, and some of the co-movement in the data. The major problem concerns the labor share in income which is not well captured. Column 1 of the model fits the data in terms of employment and unemployment behavior and has reasonable but limited success in fitting hiring flows (fits persistence and cyclicality, understates volatility). Column (4) seems to be providing the better fit for the labor share, doing relatively well on volatility, reasonably well on persistence, moderately well on co-movement with hiring and doing badly in terms of cyclical behavior.

The model also generates predictions with respect to the behavior of v and q which are unobserved in U.S. data as explained in Section 4.2 above. Strictly speaking, the moments involving these variables cannot be compared to the data. But some predictions look reasonable based on the theory and on experience in other economies: the negative correlation of u and v (the 'Beveridge curve') and persistent vacancy rates.

Summing up, no single specification matches the data on all dimensions. Focusing on the behavior of unemployment, employment, and hiring, it looks as though specification 1 (the official unemployment pool and the 'want a job' category) is the most fitting, though specification 0 (official unemployment pool) cannot be ruled out.

I turn now to look at the mechanism driving this model-data fit.

6. The underlying mechanism

The discussion up till now has shown which data specification is best explained by the model. The natural question to ask now is what underlies the fit. In order to understand the essential mechanism in operation, consider the following steady-state equation which is a re-writing of (13)[14]

$$\frac{\Theta}{\mu}\widetilde{Q}^{\gamma+1}\frac{(v/(1-u))^{\gamma}}{(v/u)^{-\sigma}} = \Phi\pi, \tag{16}$$

where $\widetilde{Q} = \phi + (1-\phi)Q$.

Eq. (16) shows the vacancy creation decision as an optimality condition equating the marginal costs of hiring (the LHS) with the asset value of the match (the RHS). It is clear that the responsiveness of vacancies (v) depends on the two elasticity parameters γ (of the hiring cost function) and σ (of the matching function). The higher is each of these, the less responsive is vacancy creation. Many studies have assumed this to be a linear function ($\gamma = 0$), thereby imposing a particular shape on the marginal cost function.

The RHS is the asset value of the match. This value can vary because per period profits π vary or because the discount factor Φ varies. The former may vary because of changes in the surplus itself or changes in the sharing of the surplus, with a key parameter being η. Any policy change in the replacement ratio τ, for example, will change η and consequently the sharing of the match surplus. Changes in the discount factor Φ can happen because of changes in productivity growth (G^X), changes in the discount factor (β), or changes in match dissolution rate (δ).

The following ingredients are therefore essential:

(i) The *shape of the hiring costs function* determining the LHS of Eq. (16) i.e. the marginal cost function. In this context γ and σ are key parameters.

[14]This equation is the relevant non-stochastic steady state equation. The discussion of the mechanism is done in terms of this equation for simplicity of exposition. Note that the full stochastic dynamic system is given by Eqs. (4)–(6), (11), and (12).

(ii) The formulation of the *match surplus*—this depends both on the data used and on all key parameters of the model.
(iii) the *surplus sharing rule*, where η is the key parameter.
(iv) the *discounting* of the match surplus—here the data used (for G^X, β and δ) and their stochastic properties are key.

What is important among these elements of the model? In order to see that I undertake some counterfactual simulations, reported in Table 7.[15]

One key element is the convexity of hiring costs, i.e., there is an important role for γ. This parameter is important because it is the main determinant of the elasticity of vacancies with respect to the present value of the match. Values of γ determine the persistence and volatility of vacancy creation. The latter then influences the second moments of matching, and consequently the moments of unemployment and employment. To see this, panel a of the table presents the outcomes when γ is set to zero, i.e., postulating linear hiring costs. The table shows that when moving from convex ($\gamma = 2$) to linear costs ($\gamma = 0$), all the persistence statistics decline, getting further away from the data; employment volatility falls from the data-consistent 0.021 (in log terms) to 0.015, (and likewise for unemployment); wages become counterfactually more volatile; hiring and separation rates become more disconnected and job finding rates become less pro-cyclical. The relationship between u and v, the Beveridge curve, turns positive, and market tightness (v/u) volatility falls by a half. The only good point is that hiring volatility comes closer to the data.

Another key element is the role played by the match dissolution or separation rate δ. As it is a variable with a relatively high mean, it is the main determinant of the relevant discount factor Φ; as it has relatively high volatility and persistence, it makes the present value of the match volatile and persistent. This in turn engenders the volatility and persistence of vacancies, hiring, and unemployment. To see this point consider panel b of Table 7, where, in the column labeled counterfactual 1, the AR coefficient of δ is reduced to 0.1, counterfactually. The change in the moments is substantial: the persistence of all variables falls from 0.98 to 0.55–0.76 and the volatility of all variables is reduced so that standard deviations are 2–5% of their benchmark, actual values. The co-movement statistics weaken: Most of the co-movement relations weaken moderately, and the negative co-movement relations of the labor share with the hiring rate and of unemployment with vacancies weaken substantially.

I reset the persistence to its actual value, and, in the column labeled counterfactual 2, I set the variance and co-variance of δ to zero. This dramatically lowers the persistence, the volatility, and the co-movement statistics of all the variables.

Next, I examine whether the interest rate has a similar effect via β. The column labeled counter-factual 3 sets its variance and co-variance to zero but this change hardly has any effect.

Finally, in panel c, I combine both a linear hiring cost function ($\gamma = 0$) and zero variance and co-variance for β and δ. The results are negative auto-correlation statistics, very low volatility, weaker co-movement, and a positive, rather than negative, correlation between u and v. Comparing panel c to panels a and b, one can see that allowing no variance for δ is the dominant effect on all moments.

[15] Note that the simulations pertain to the full stochastic dynamic system, while Eq. (16) is the non-stochastic steady state of this system.

Table 7

Hiring costs	Data	Benchmark	Counter-factual
		Convex $\gamma = 2$	Linear $\gamma = 0$
a. Convexity of hiring costs			
$\rho(\widehat{u}_t, \widehat{u}_{t-1})$	0.971	0.983	0.881
$\rho(\widehat{m}_t, \widehat{m}_{t-1})$	0.853	0.986	0.467
$\rho(\widehat{s}_t, \widehat{s}_{t-1})$	0.884	0.976	0.592
$std(\widehat{n}_t)$	0.022	0.021	0.015
$std(\widehat{u}_t)$	0.188	0.183	0.126
$std(\widehat{m}_t)$	0.085	0.052	0.090
$std(\widehat{\frac{v_t}{u_t}})$	–	0.219	0.103
$std(\widehat{s}_t)$	0.016	0.056	0.068
$\rho(\widehat{u}_t, \widehat{m}_t)$	0.810	0.997	0.888
$\rho(\widehat{u}_t, \widehat{P}_t)$	−0.933	−1.00	−0.743
$\rho(\widehat{n}_t, \widehat{s}_t)$	−0.160	0.997	0.936
$\rho(\widehat{m}_t, \widehat{s}_t)$	0.451	−0.989	−0.993
$\rho(\widehat{m}_t, \widehat{\delta}_t)$	0.908	0.862	0.599
$\rho(\widehat{u}_t, \widehat{v}_t)$	–	−0.985	0.580

	Data	Benchmark	Counterfactual 1	Counterfactual 2	Counterfactual 3
			δ AR $= 0.1$	$\sigma_\delta = 0$	$\sigma_\beta = 0$
b. The interest rate and the separation rate					
$\rho(\widehat{u}_t, \widehat{u}_{t-1})$	0.971	0.983	0.763	0.704	0.983
$\rho(\widehat{m}_t, \widehat{m}_{t-1})$	0.853	0.986	0.696	−0.152	0.986
$\rho(\widehat{s}_t, \widehat{s}_{t-1})$	0.884	0.976	0.553	0.262	0.977
$std(\widehat{n}_t)$	0.022	0.021	0.001	0.00006	0.021
$std(\widehat{u}_t)$	0.188	0.183	0.004	0.0006	0.183
$std(\widehat{m}_t)$	0.085	0.052	0.001	0.0005	0.051
$std(\widehat{\frac{v_t}{u_t}})$	–	0.219	0.006	0.001	0.219
$std(\widehat{s}_t)$	0.016	0.056	0.002	0.0006	0.056
$\rho(\widehat{u}_t, \widehat{m}_t)$	0.810	0.997	0.886	0.279	0.997
$\rho(\widehat{u}_t, \widehat{P}_t)$	−0.933	−1.00	−0.984	−0.620	−1.00
$\rho(\widehat{n}_t, \widehat{s}_t)$	−0.160	0.997	0.917	0.304	0.997
$\rho(\widehat{m}_t, \widehat{s}_t)$	0.451	−0.989	−0.627	0.830	−0.989
$\rho(\widehat{m}_t, \widehat{\delta}_t)$	0.908	0.862	0.304	–	0.862
$\rho(\widehat{u}_t, \widehat{v}_t)$	–	−0.985	−0.733	−0.140	−0.985

	Data	Benchmark	Counter-factual
			$\gamma = 0; \sigma_\delta = \sigma_\beta = 0$
c. Convexity and the separation rate			
$\rho(\widehat{u}_t, \widehat{u}_{t-1})$	0.971	0.983	−0.118
$\rho(\widehat{m}_t, \widehat{m}_{t-1})$	0.853	0.986	−0.556
$\rho(\widehat{s}_t, \widehat{s}_{t-1})$	0.884	0.976	−0.556
$std(\widehat{n}_t)$	0.022	0.021	0.00004
$std(\widehat{u}_t)$	0.188	0.183	0.00004
$std(\widehat{m}_t)$	0.085	0.052	0.0007
$std(\widehat{\frac{v_t}{u_t}})$	–	0.219	0.0008
$std(\widehat{s}_t)$	0.016	0.056	0.0004

Table 7 (*continued*)

	Data	Benchmark	Counter-factual
			$\gamma = 0; \sigma_\delta = \sigma_\beta = 0$
$\rho(\widehat{u}_t, \widehat{m}_t)$	0.810	0.997	0.720
$\rho(\widehat{u}_t, \widehat{P}_t)$	−0.933	−1.00	0.255
$\rho(\widehat{n}_t, \widehat{s}_t)$	−0.160	0.997	0.773
$\rho(\widehat{m}_t, \widehat{s}_t)$	0.451	−0.989	−0.997
$\rho(\widehat{m}_t, \widehat{\delta}_t)$	0.908	0.862	–
$\rho(\widehat{u}_t, \widehat{v}_t)$	–	0.985	0.588

Thus the performance of the model hinges to a large extent on the formulation of γ and on the stochastic properties of δ. This point is further investigated in Yashiv (2006b). The lack of fit in part of the literature is due to the use of a linear hiring cost function ($\gamma = 0$) instead of a convex one ($\gamma = 2$ here), and due to different stochastic properties assigned to the separation rate δ.

Other elements of the model play a smaller role. The wage parameter η basically has a scale effect on per period profits and hence on the scale of asset values. It therefore affects the value of the variables at the steady state but does not affect the dynamics, as it does not affect the response of vacancy creation to asset values. The matching function parameter σ, that does have this 'elasticity' type of effect, has a range of possible variation that is much smaller than the variation in values of γ. For example, a reasonable change in σ would be 0.1 or 0.2 relative to the benchmark value (which is 0.4), but a move from linear ($\gamma = 0$) to cubic ($\gamma = 2$) costs is a change of 2 in the value of γ The interest rate and the rate of productivity growth in their turn play a much smaller role than the separation rate in discounting future values. While δ has a sample mean of 8.6% and a standard deviation of 0.8%, the rate of productivity growth ($G^X - 1$) has a sample mean of 0.4% and standard deviation of 0.6%. The rate of interest ($(1/\beta) - 1$) has a sample mean of 1.4% and a relatively high standard deviation, 5.5%, but as shown in panel b of Table 7 it does not play a significant role.

The afore-going discussion implies the following modelling lessons for the aggregate U.S. labor market:

(i) In terms of the model formulation, two ingredients are important: convexity of the hiring costs function and allowing the separation rate to affect the dynamics, in addition to, and contemporaneously with, any other shock, such as productivity shocks.
(ii) In terms of U.S. data, from amongst the alternative pools of searching workers, the one consisting of official unemployment and the 'want a job now' category seems to be the most consistent with the model.

Finally, the relation of these results with some recent papers merits discussion. Cole and Rogerson (1999) found that the model can account for business cycle facts only if the average duration of unemployment is relatively high (9 months or longer) and substantially longer than average duration in BLS data on the unemployed. This would be consistent with the broader specifications of the unemployment pool discussed above (see Fig. 3 and Table 3).

Shimer (2005) showed that in the standard matching model productivity shocks of empirically plausible magnitude cannot generate the observed, large cyclical fluctuations in unemployment and vacancies.[16] The key reason for this result is that the standard model assumes that wages are determined by Nash bargaining, which in turn implies that wages are "too flexible." For example, following a positive productivity shock, wages increase, absorbing the shock, thereby dampening the incentives of firms to create new jobs. In particular, while the model predicts roughly the same volatility of the vacancy to unemployment ratio and of productivity, the data indicate this ratio is actually 18 times more volatile. The results here are different, showing that the model does fit most of the facts, albeit not wage behavior. The reasons for this difference are the modelling of convex, rather than linear, hiring costs, and the stochastic structure assumed for the separation rate.

Building upon the Shimer (2005) results, Hall (2005) claims that explaining the job finding rate is key for the understanding of business cycles, and that the separation rate is roughly constant. He suggests that one needs to specify some form of wage stickiness to fit the data. The current paper shows that the standard model can account for the behavior of the job finding rate, and suggests that the separation rate plays an important role as a discount rate. It does find, though, that wage behavior is not well explained by the standard model.

Fujita (2004) conducted empirical tests showing that vacancies are much more persistent in the data than the low persistence implied by the model. The results here suggest that with convex hiring costs this problem is remedied.

7. Conclusions

The paper has formulated a model of U.S. aggregate labor market dynamics using a log-linear approximation of the standard search and matching model. It has looked at alternative formulations of the data that would be consistent with the concepts of the model. Using a VAR of the actual data it injected driving shocks into the model. Comparing the resulting moments implied by the model to the moments in U.S. data, it has shown that it can account for much of observed labor market fluctuations. In particular, the model fits the data on persistence and volatility of most variables, on the negative relationship between vacancies and unemployment, and on the pro-cyclicality of the workers' job-finding rate. For the same formulation of the data, it fails to capture the a-cyclicality of the labor share and its moderate positive co-variation with hiring, and it understates the latter's volatility; it is, however, able to come closer to capturing these features given a different configuration of the data.

The analysis has produced an empirically grounded version of the search and matching model—complete with parameter values and data series—that can be used to study the U.S. labor market, including policy questions.

The paper raises a number of issues for further research. A key one is the need to further explain the mechanisms in operation. In particular, one may ask what is the role of search and matching frictions in the dynamics; for example, how would different degrees of

[16] Veracierto (2002) has also shown that the model fails to fit the volatility and cyclicality of unemployment and employment.

frictions lead to different outcomes. This issue is the subject of current research (see Yashiv, 2006b). Other issues are: do the results carry over to other economies and, if so, what are the cross-country differences in parameter values and in the dynamics? For example, it would be of interest to see whether such differences can explain the different U.S.—European unemployment experiences.

Acknowledgements

Prepared for the conference in honor of Dale Mortensen at Sandbjerg, Denmark, August 15–18, 2004. I am indebted to Dale for inspiration and encouragement over the years. I am grateful to the editor Zvi Eckstein and to two anonymous referees for useful comments, to Jordi Gali , Martin Lettau, Fabien Postel-Vinay and Etienne Wasmer for useful conversations, to Craig Burnside for advice, data and software, to Jeffrey Fuhrer, Hoyt Bleakley, Ann Ferris and Elizabeth Walat for data, and to Darina Waisman for excellent research assistance. Any errors are mine.

Appendix A. Data: Sources and definitions

All data are quarterly U.S. data for the period 1970:I–2003:IV, except for hires and separations which begin in 1976:I and end in 2003:III.

Variable	Symbol	Source
Unemployment – official pool	U^0	CPS, BLS series id: LNS13000000
Workers out of l.f. who "want a job"	WAJ	CPS, BLS series id: LNU05026639
Unemployment – additional pools (see Table 1 below)	U	CPS, BLS
Employment (total), household survey	N	CPS, BLS series id: LNS12000000
Vacancies – Index of Help Wanted ads	V	Conference Board[a]
Hires	QV	CPS, Boston Fed computations[b]
Separations	δN	CPS, Boston Fed computations[b]
Working age population[c]	POP	CPS, BLS series id: LNU00000000
Labor share[d]	$s = WN/F$	Table 1.12. NIPA, BEA[e]
Productivity	F/N	BLS
Cost of finance (equity and debt)[f]	r	Tables 1.1.5; 1.1.6 NIPA, BEA

Notes: BLS series are taken from http://www.bls.gov/cps/home.htm

[a]Data were downloaded from Federal Reserve Bank of St. Louis http://research.stlouisfed.org/fred2/series/HELPWANT/10.

[b]See Bleakley et al. (1999) for construction methodology. I thank Jeffrey Fuhrer and Elizabeth Walat for their work on this series.

[c]Total civilian noninstitutional population 16 years and older.

[d]Total compensation of employees divided by GDP.

[e]http://www.bea.doc.gov/bea/dn/home/gdp.htm

[f]This is a weighted average of the returns to debt and equity, with the share of debt finance as reported in Fama and French (1999), Journal of Finance, LIV, pp. 1939–1967.

References

Abraham, K.G., Spletzer, J.R., Stewart, J.C., 1999. Why do different wage series tell different stories? American Economic Review, AEA Papers and Proceedings 89 (2), 34–39.

Anderson, P.M., Meyer, B.D., 1997. Unemployment insurance takeup rates and the after tax value of benefits. Quarterly Journal of Economics CXII, 913–937.

Andolfatto, D., 1996. Business cycles and labor market search. American Economic Review 86, 112–132.

Blanchard, O.J., Diamond, P., 1989. The Beveridge curve. Brookings Papers on Economic Activity 1, 1–60.

Bleakley, H., Ferris, A.E., Fuhrer, J.C., 1999. New data on worker flows during business cycles. New England Economic Review July–August, 49–76.

Burdett, K., Cunningham, E.J., 1998. Toward a theory of vacancies. Journal of Labor Economics 16 (3), 445–478.

Burnside, A.C., 1997. Notes on the Linearization and GMM Estimation of Real Business Cycle Models. Mimeo.

Cahuc, P., Marque, F., Wasmer, E., 2004. A theory of wages and labor demand with intrafirm bargaining and matching frictions. Mimeo.

Christiano, L.J., Eichenbaum, M., Vigfusson, R., 2003. What happens after a technology shock? NBER Working Paper No. 9819.

Clark, K.B., Summers, L.H., 1979. Labor market dynamics and unemployment: A reconsideration. Brookings Papers on Economic Activity 1, 13–60.

Cole, H.L., Rogerson, R., 1999. Can the Mortensen–Pissarides matching model match the business cycle facts? International Economic Review 40 (4), 933–960.

Diamond, P.A., 1982. Wage determination and efficiency in search equilibrium. Review of Economic Studies 49, 761–782.

Fujita, S., 2004. Vacancy persistence. FRB of Philadelphia Working Paper No. 04-23.

Hall, R.E., 2005. Job loss, job finding, and unemployment in the U.S. economy over the past fifty years. NBER Macroeconomics Annual, forthcoming.

Jones, S.R.G., Riddell, W.C., 2000. The Dynamics of U.S. Labor Force Attachment. Mimeo.

Krueger, A.B., 1999. Measuring labor's share. American Economic Review, AEA Papers and Proceedings 89 (2), 45–51.

Merz, M., 1995. Search in the labor market and the real business cycle. Journal of Monetary Economics 36, 269–300.

Merz, M., Yashiv, E., 2006. Labor and the market value of the firm. American Economic Review, forthcoming.

Mortensen, D.T., 1970. A theory of wage and employment dynamics. In: Phelps, E.S., et al. (Eds.), The Microeconomic Foundations of Employment and Inflation Theory. Norton, New York.

Mortensen, D.T., 1978. Specific capital and labor turnover. Bell Journal of Economics 9, 572–586.

Mortensen, D.T., 1982a. The matching process as a non-cooperative bargaining game. In: McCall, J.J. (Ed.), The Economics of Information and Uncertainty. University of Chicago Press, New York.

Mortensen, D.T., 1982b. Property rights and efficiency in mating, racing and related games. American Economic Review 72, 968–979.

Mortensen, D.T., Pissarides, C.A., 1994. Job creation and job destruction in the theory of unemployment. Review of Economic Studies 61, 397–415.

Mortensen, D.T., Pissarides, C.A., 1999. Job reallocation, employment fluctuations, and unemployment. in: John B. Taylor, Michael Woodford (Eds.), Handbook of Macroeconomics, vol. 1B, North-Holland, Amsterdam, pp. 1171–1228 (Chapter 18).

Petrongolo, B., Pissarides, C.A., 2001. Looking into the black box: a survey of the matching function. Journal of Economic Literature 39 (2), 390–431.

Pissarides, C.A., 2000. Equilibrium Unemployment Theory, second ed. MIT Press, Cambridge.

Shimer, R., 2005. The cyclical behavior of equilibrium unemployment and vacancies: evidence and theory. American Economic Review 95 (1), 25–49.

Veracierto, M., 2002. On the cyclical behavior of employment, unemployment and labor force participation. WP 2002-12, Federal Reserve Bank of Chicago.

Yashiv, E., 2000a. The determinants of equilibrium unemployment. American Economic Review 90 (5), 1297–1322.

Yashiv, E., 2000b. Hiring as investment behavior. Review of Economic Dynamics 3, 486–522.

Yashiv, E., 2004. Macroeconomic policy lessons of labor market frictions. European Economic Review 48, 259–284.

Yashiv, E., 2006a. Search and matching in macroeconomics. CEPR Discussion Paper 5459, European Economic Review, forthcoming.

Yashiv, E., 2006b. Forward looking hiring behavior and the dynamics of the aggregate labor market. Available at ⟨http://www.tau.ac.il/~yashiv/⟩.

Yashiv, E., 2006c. Evaluating the performance of the search and matching model. in: H. Bunzel, B.J. Christensen, G.R. Neumann, and J.M. Robin (Eds.), Structural Models of Wage and Employment Dynamics. Elsevier, North Holland, Amsterdam, forthcoming, pp. 511–544 (Chapter 20).