

Supplement to "The importance of hiring frictions in business cycles"

31-10-2021

RENATO FACCINI
Danmarks Nationalbank, CFM(LSE)

ERAN YASHIV
Tel Aviv University, CFM(LSE), CEPR

APPENDIX A: THE EXTENDED MODEL

In this Appendix we fully elaborate on the extended model and on the further explorations of its mechanism.

Sub-Section [A.1](#) presents the extended model, which is essentially a medium-scale general equilibrium model, catering for a richer framework.

We have presented in Sub-Section [6.2](#) above the full impulse response functions of this extended model, revisiting the mechanism discussed above.

Two sub-sections then examine the role of our formulation of hiring costs: in sub-section [A.2](#) we look at output costs vs pecuniary costs and in sub-section [A.3](#) we look at internal vs external costs. Finally, sub-Section [A.5](#) reports on the robustness of the results to variations in the Taylor rule.

A.1 The extended model

The model augments the simple set-up of Section [4](#) to specifically include a matching function in the labor market, external habits in consumption and investment adjustment costs to the problem of the households, external hiring costs, trend inflation and inflation indexation in the problem of the intermediate firms, and exogenous wage rigidity in the wage rule.

Renato Faccini: rmmf@nationalbanken.dk

Eran Yashiv: yashiv@tauex.tau.ac.il

A.1.1 *Households* Let $\vartheta \in [0, 1)$ be the parameter governing external habit formation. The intertemporal problem of a household indexed by subscript j is to maximize the discounted present value of current and future utility:

$$\max_{\{C_{t+s,j}, I_{t+s,j}, B_{t+s+1,j}\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \beta^s \left[\ln(C_{t+s,j} - \vartheta C_{t+s-1}) - \frac{\chi}{1+\varphi} N_{t+s,j}^{1+\varphi} \right],$$

subject to the budget constraint (2) and the laws of motion for employment (3) and capital:

$$K_{t,j} = (1 - \delta_K)K_{t-1,j} + \left[1 - S\left(\frac{I_{t,j}}{I_{t-1,j}}\right) \right] I_{t,j}, \quad 0 \leq \delta_K \leq 1, \quad (1)$$

where S is the investment adjustment cost function. It is assumed that $S(1) = S'(1) = 0$, and $S''(1) \equiv \phi > 0$. Denoting by λ_t the Lagrange multiplier associated with the budget constraint, and by Q_t^K the Lagrange multiplier associated with the law of motion for capital, under the assumption that all households are identical in equilibrium, the conditions for dynamic optimality are:

$$\lambda_t = \frac{1}{P_t(C_t - \vartheta C_{t-1})},$$

$$\frac{1}{R_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}, \quad (2)$$

$$Q_t^K = E_t \Lambda_{t,t+1} \left[\frac{X_{t+1}^K}{P_{t+1}} + (1 - \delta_K) Q_{t+1}^K \right] \quad (3)$$

where $\Lambda_{t,t+1} = \frac{P_{t+1}}{R_t}$.

$$V_t^N = \frac{W_t}{P_t} - \frac{\chi N_t^\varphi}{\lambda_t P_t} - \frac{x_t}{1 - x_t} V_t^N + E_t \Lambda_{t,t+1} (1 - \delta_N) V_{t+1}^N, \quad (4)$$

and

$$Q_t^K \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} \right] + E_t \Lambda_{t,t+1} Q_{t+1}^K S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 = 1, \quad (5)$$

where the Euler equation (2), the value of capital (3), and the value of a marginal job to the household (4) correspond to equations (5), (17) and (7) in the simple model of Section 4, respectively.

A.1.2 *Intermediate firms* We assume price stickiness *à la* Rotemberg (1982), meaning firms maximize current and expected discounted profits subject to quadratic price adjustment costs. We assume that adjustment costs depend on the ratio between the new reset price and the one set in the previous period, adjusted by a geometric average of gross steady state inflation, $1 + \bar{\pi}$, and past inflation. We denote by ψ the parameter that captures the degree of indexation to past inflation.

Firms maximize the following expression:

$$\max_{\{P_{t+s,i}, H_{t+s,i}, K_{t+s,i}\}} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left\{ \frac{P_{t+s,i}}{P_{t+s}} Y_{t+s,i} - \frac{W_{t+s,i}}{P_{t+s}} N_{t+s,i} - \frac{X_{t+s}^K}{P_{t+s}} K_{t+s,i} - \frac{\zeta}{2} \left(\frac{P_{t+s,i}}{(1 + \pi_{t+s-1})^\psi (1 + \bar{\pi})^{1-\psi} P_{t+s-1,i}} - 1 \right)^2 Y_{t+s} \right\} \quad (6)$$

where $\Lambda_{t,t+s}$, defined above, is the real discount factor of the households who own the firms, taking as given the demand function (8) and subject to the law of motion for employment (12) and the constraint that output equals demand:

$$\left(\frac{P_{t,i}}{P_t} \right)^{-\epsilon} Y_t = f(A_t, N_{t,i}, K_{t,i}) [1 - \tilde{g}(V_{t,i}, H_{t,i}, N_{t,i})]. \quad (7)$$

To ensure comparability with a literature that has modelled hiring costs predominantly as vacancy posting costs, we follow Sala, Soderstrom, and Trigari (2013), and assume that the fraction of output forgone due to hiring activities is given by the hybrid function:

$$\tilde{g}_{t,i} = \frac{e}{2} \left(\frac{V_{t,i}}{N_{t,i}} \right)^{\eta^q} \left(\frac{H_{t,i}}{N_{t,i}} \right)^{2-\eta^q}. \quad (8)$$

When $\eta^q = 0$ this function reduces to

$$\tilde{g}_{t,i} = \frac{e}{2} \left(\frac{H_{t,i}}{N_{t,i}} \right)^2,$$

which is the same expression as (10), where all friction costs depend on the firm-level hiring rate and are not associated with the number of vacancies per se. In this case, marginal hiring costs are not affected by the probability that a vacancy is filled. When

instead $\eta^q = 2$ the function becomes

$$\tilde{g}_t = \frac{e}{2} \left(\frac{V_{t,i}}{N_{t,i}} \right)^2,$$

and is only associated with posting vacancies.

It can be easily shown that equation (8) implies that an increase in the vacancy filling rate q_t decreases the marginal cost of hiring.¹ For intermediate values of $\eta^q \in (0, 2)$, the specification in (8) allows for both hiring rates and vacancy rates to matter for the costs of hiring in different proportions.

Following a similar argument to the one proposed by Gertler, Sala and Trigari (2008), we note that by choosing vacancies, the firm directly controls the total number of hires $H_{t,i} = q_t V_{t,i}$, since it knows the vacancy filling rate q_t . Hence, $H_{t,i}$ can be treated as a control variable.

The optimality conditions with respect to $H_{t,i}$, $N_{t,i}$, $K_{t,i}$ and $P_{t,i}$ are:

$$Q_t^N = \Psi_t g_{H,t}, \quad (9)$$

$$Q_t^N = \Psi_t (f_{N,t} - g_{N,t}) - \frac{W_t}{P_t} + (1 - \delta_N) E_t \Lambda_{t,t+1} Q_{t+1}^N, \quad (10)$$

$$\frac{X_t^K}{P_t} = \Psi_t (f_{K,t} - g_{K,t}) \quad (11)$$

and

$$\begin{aligned} & (1 - \epsilon) \left(\frac{P_{t,i}}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} + \Psi_t \epsilon \left(\frac{P_{t,i}}{P_t} \right)^{-\epsilon-1} \frac{Y_t}{P_t} \\ & - \zeta \left(\frac{P_{t,i}}{(1 + \pi_{t-1})^\psi (1 + \bar{\pi})^{1-\psi} P_{t-1,i}} - 1 \right) \frac{Y_t}{(1 + \pi_{t-1})^\psi (1 + \bar{\pi})^{1-\psi} P_{t-1,i}} \\ & + E_t \Lambda_{t,t+1} \zeta \left(\frac{P_{t+1,i}}{(1 + \pi_t)^\psi (1 + \bar{\pi})^{1-\psi} P_{t,i}} - 1 \right) Y_{t+1} \left(\frac{P_{t+1,i}}{\left((1 + \pi_{t-1})^\psi (1 + \bar{\pi})^{1-\psi} P_{t,i} \right)^2} \right) = 0. \end{aligned}$$

¹Equation (8) can be rewritten as $\tilde{g}_{t,i} = \frac{e}{2} q_t^{-\eta^q} \left(\frac{H_{t,i}}{N_{t,i}} \right)^2$, which implies that $g_{H,t,i} = \frac{\partial}{\partial H_{t,i}} \tilde{g}_{t,i} f_{t,i} = e q_t^{-\eta^q} \frac{H_{t,i}}{N_{t,i}^2} f_{t,i}$.

Since all firms set the same price and therefore produce the same output in equilibrium, the above equation can be rearranged as follows:

$$\left(\frac{1 + \pi_t}{(1 + \pi_{t-1})^\psi (1 + \bar{\pi})^{1-\psi}} - 1 \right) \frac{1 + \pi_t}{(1 + \pi_{t-1})^\psi (1 + \bar{\pi})^{1-\psi}} = \frac{1 - \epsilon}{\zeta} + \frac{\epsilon}{\zeta} \Psi_t$$

$$+ E_t \frac{1}{R_t / (1 + \pi_{t+1})} \left[\left(\frac{1 + \pi_{t+1}}{(1 + \pi_t)^\psi (1 + \bar{\pi})^{1-\psi}} - 1 \right) \frac{1 + \pi_{t+1}}{(1 + \pi_t)^\psi (1 + \bar{\pi})^{1-\psi}} \frac{Y_{t+1}}{Y_t} \right]. \quad (12)$$

Merging the FOCs for capital of households and firms (3) and (11) we get:

$$Q_t^K = E_t \Lambda_{t,t+1} \left[\Psi_{t+1} (f_{K,t+1} - g_{K,t+1}) + (1 - \delta_K) Q_{t+1}^K \right] \quad (13)$$

A.1.3 *Matching* We now assume that in the labor market, unemployed workers

and vacancies come together through the constant returns to scale matching function

$$H_t = \frac{U_{0,t} V_t}{\left(U_{0,t}^l + V_t^l \right)^{\frac{1}{l}}}, \quad (14)$$

where H_t denotes the number of matches, or hires, V_t aggregate vacancies, $U_{0,t}$ the aggregate measure of workers who are unemployed at the beginning of each period t , and l is a parameter. This matching function was used by Den Haan, Ramey, and Watson (2000) and ensures that the matching rates for both workers and firms are bounded above by one. We denote the job finding rate by $x_t = \frac{H_t}{U_{0,t}}$ and the vacancy filling rate by $q_t = \frac{H_t}{V_t}$.

A.1.4 *Wage norm* We assume wage rigidity in the form of a Hall (2005) type wage norm:

$$\frac{W_t}{P_t} = \omega \frac{W_{t-1}}{P_{t-1}} + (1 - \omega) \frac{W_t^{NASH}}{P_t}, \quad (15)$$

where ω is a parameter governing real wage stickiness, and W_t^{NASH} denotes the Nash reference wage

$$\frac{W_t^{NASH}}{P_t} = \arg \max \left\{ \left(V_t^N \right)^\gamma \left(Q_t^N \right)^{1-\gamma} \right\}, \quad (16)$$

which yields

$$\frac{W_t^{NASH}}{P_t} = \gamma \Psi_t (f_{N,t} - g_{N,t}) + (1 - \gamma) \left[\chi N_t^\varphi (C_t - \vartheta C_{t-1}) + \frac{x_t}{1 - x_t} \frac{\gamma}{1 - \gamma} Q_t^N \right]. \quad (17)$$

A.1.5 *Final good firms* Final firms maximize

$$\max P_t Y_t - \int_0^1 P_{t,i} Y_{t,i} di$$

subject to

$$Y_t = \left(\int_0^1 Y_{t,i}^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}.$$

Taking first order conditions with respect to Y_t and $Y_{t,i}$ and merging we can solve for the demand function

$$Y_{t,i} = \left(\frac{P_{t,i}}{P_t} \right)^{-\epsilon} Y_t. \quad (18)$$

A.1.6 *The monetary and fiscal authorities and market clearing* The model is closed by assuming that the government runs a balanced budget, as per equation (21), the monetary authority follows the Taylor rule in equation (22), the goods market clears as per equation (23) and the capital market clears, i.e. $\int_{i=0}^1 K_{t,i} di = \int_{j=0}^1 K_{t-1,j} dj$, where i and j index firms and households, respectively.

A.1.7 *Calibration* The model is calibrated following the same steps as in Sub-Section 5.1. The parameter values for the friction cost scale parameter e and the bargaining power γ are set so as to hit the same targets as in the calibration of the simple model. The parameter of the matching function l is calibrated to target a vacancy filling rate (q) of 70%, as in Den Haan, Ramey and Watson (2000). The scale parameter in the utility function χ is no longer normalized to equal one, but is set so as to target the same replacement ratio of the opportunity cost of work over the marginal revenue product (77%), as implied by the benchmark calibration in Sub-Section 5.1. All other parameter values that are common to the simple model are set to the same value reported in Table 3. As for the new parameters, the investment adjustment cost parameter ϕ is set to 2.5, and the habit parameter to $\vartheta = 0.8$, reflecting the estimate by Christiano, Eichenbaum and Trabandt (2016). The parameter governing trend inflation is set to $\bar{\pi} = 0.783\%$, which corresponds to the

average of the US GDP deflator over the calibration period. Given that, the value of the discount factor β , is set so as to match a 1% nominal rate of interest. We set the degree of indexation to a moderate value of $\psi = 0.5$, and the parameter governing wage rigidity to $\omega = 0.87$, in order to match the persistence of the US real wage data. Finally, we set the elasticity of the hiring friction function η^q to 0.49, which is value estimated by Sala, Soderstrom, and Trigari (2013) for the US economy. We note that this estimate implies a stronger influence of vacancy filling rates in hiring costs than what would be implied by the micro-evidence reported by Silva and Toledo (2009), which would map into a coefficient of η^q of 0.145. In the relatively low friction benchmark, the parameter e governing the scale of hiring frictions is set following the same strategy as in Section 5.1: the value of e is set to 1.2 so as to target a ratio of marginal hiring costs to productivity of 0.20. To inspect the mechanism, we will also report impulse responses for a relatively high frictions benchmark, where the scale of the hiring costs function is raised to 5, in order to match the empirical evidence in Silva and Toledo (2009), where average hiring costs are equal to 55% of quarterly wages.

Parameter values and calibration targets for the extended model are reported in Table A-1.

TABLE A1. Calibrated parameters and steady state values, extended model

<i>Panel A: Parameters</i>		
<i>Description</i>	<i>Parameter</i>	<i>Value</i>
Discount factor	β	0.9978
Separation rate	δ_N	0.126
Capital depreciation rate	δ_K	0.024
Elasticity of output to labor input	α	0.66
Hiring friction scale parameter	e	1.2
Elasticity of hiring costs to vacancy filling rate	η^q	0.49
Elasticity of substitution	ϵ	11
Workers' bargaining power	γ	0.4725
Scale parameter in utility function	χ	5.44
Inverse Frisch elasticity	φ	4
Matching function parameter	l	1.42
Price frictions (Rotemberg)	ζ	120
External habits	ϑ	0.8
Exogenous wage rigidity	ω	0.87
Investment adjustment costs	ϕ	2.5
Trend inflation	$\bar{\pi}$	0.783
Inflation indexation	ψ	0.5
Taylor rule coefficient on inflation	r_π	1.5
Taylor rule coefficient on output	r_y	0.125
Taylor rule smoothing parameter	ρ_r	0.75
Autocorrelation technology shock	ρ_a	0.95
Autocorrelation monetary shock	ρ_ξ	0
<i>Panel B: Steady State Values</i>		
<i>Definition</i>	<i>Expression</i>	<i>Value</i>
Total adjustment cost/ net output	$g / (f - g)$	0.011
Marginal hiring cost	$g_H / [(f - g) / N]$	0.20
Marginal hiring cost/ wage	$\Psi g_H / \left(\frac{W}{P}\right)$	0.30
Average hiring cost/ wage	$\frac{g}{H} \Psi / \left(\frac{W}{P}\right)$	0.13
Opportunity cost of work/ marginal revenue prod.	$\frac{\chi C(1-\vartheta)N^\varphi}{mc(f_N - g_N)}$	0.70
Unemployment rate	u	0.111

A.2 Output costs vs. pecuniary costs of hiring

So far we have assumed that the hiring costs specified in equation (8) are expressed in units of (forgone) output. Alternatively we could have assumed, following the convention in the literature, that hiring costs are pecuniary, meaning that they are specified in units of the composite good. In this case the production function (9) is simply $Y_{t,i} = f(A_t, N_{t,i}, K_{t,i})$, and the maximization problem of the firm becomes

$$\max_{P_{t+s,i}, H_{t+s,i}, \check{K}_{t+s,i}} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left\{ \frac{P_{t+s,i}}{P_{t+s}} Y_{t+s,i} - \frac{W_{t+s} N_{t+s,i}}{P_{t+s}} - \frac{X_{t+s}^K}{P_{t+s}} K_{t+s,i} - g_{t,i} - \frac{\zeta}{2} \left(\frac{P_{t+s,i}}{\bar{P}_{t+s-1,i}} - 1 \right)^2 Y_{t+s} \right\},$$

where $g_{t,i} = \tilde{g}_{t,i} Y_{t,i}$, subject to the demand function (8), the law of motion for employment (12), and the technology constraint (13).

The main implication of assuming pecuniary costs is that the first order condition for hiring becomes:

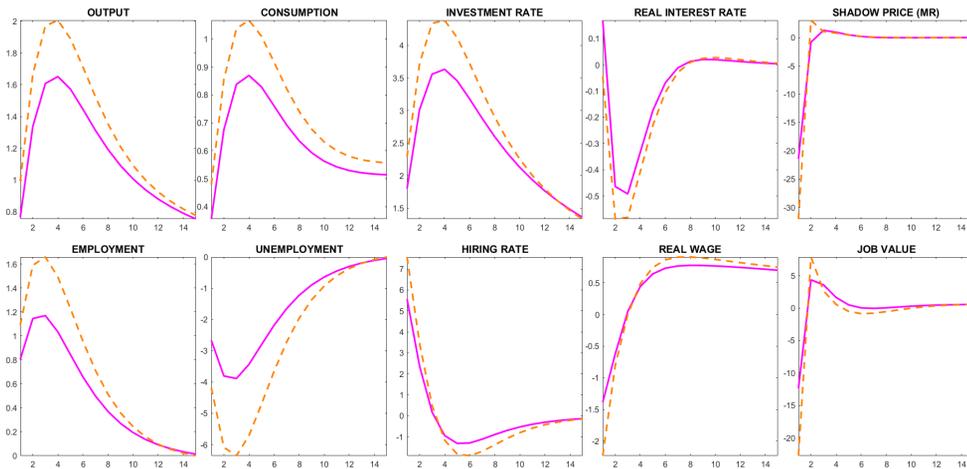
$$Q_t^N = g_{H,t},$$

which implies that the cost of the marginal hire is no longer affected directly by the shadow price Ψ_t .

This model with pecuniary costs does not generate reversals of the NK outcomes, unlike the model with output-costs. The role of hiring frictions then, is to smooth impulse responses, with negligible effects if frictions are calibrated to reflect only vacancy costs (as in Galí (2011), for example).

Interestingly, we find that the model with pecuniary costs of hiring is prone to indeterminacy even for moderate values of hiring frictions. Specifically, for the parameter vector underlying our “high” hiring cost calibration, which underpins the orange lines in Figures 3 to 5, the model with pecuniary costs does not satisfy the conditions for determinacy. The intuition for this indeterminacy is as follows. If firms expect aggregate demand to be high, they will hire more workers to increase production and meet this high level of demand. If prices are sticky and hiring costs are pecuniary, i.e., they are purchases of the composite good, the increase in the demand for hiring services stimulates aggregate demand. Hence, expectations of higher demand become self-fulfilling. If hiring costs are forgone output instead, higher hiring does not stimulate demand, and the model is less prone to indeterminacy. This implies that the conventional modelling of hiring costs as pecuniary costs, can only support equilibria where hiring frictions are sufficiently small. Thus, any estimation of such friction costs in general equilibrium can only deliver quantitatively small estimates.

FIGURE A1. Impulse responses to a positive technology shock, vacancy costs only vs vacancy and hiring costs.



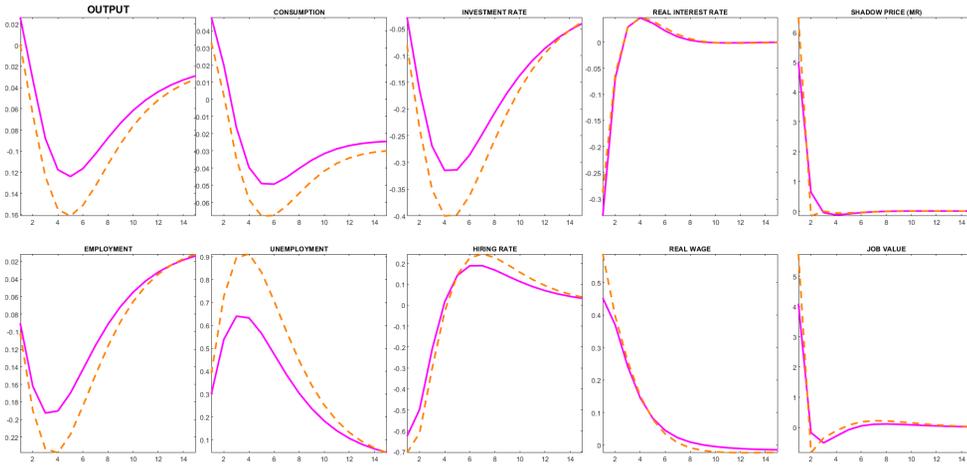
Note: Impulse responses to a 1 percent positive technology shock obtained for two different parameterizations of η^q , both with "high" hiring costs $e = 5$. The orange (dashed) line uses the benchmark $\eta^q = 0.49$, implying the co-existence of both vacancy and hiring costs; and the purple (solid) line uses $\eta^q = 2$, implying vacancy costs only. All variables are expressed in percent deviations, except hiring, investment, and real rates, which are expressed in percentage points deviations. Output is specified net of hiring costs.

A.3 Internal vs. external costs of hiring

The medium-scale model considered so far allows for both external and internal costs to affect the propagation of shocks. Here we show how this propagation changes when we exclude internal costs altogether. This exercise is convenient to relate to a literature, which has predominantly focussed on external costs of hiring. Namely, we report the impulse responses obtained under the "high" friction cost parameterization, comparing the benchmark case of $\eta^q = 0.49$ with the case of $\eta^q = 2$, which implies that hiring frictions are entirely driven by external vacancy rates. The results are shown in Figures A-1 and A-2 for technology shocks and monetary policy shocks, respectively.

The figures show that the offset to the standard NK propagation produced by our mechanism is considerably diluted in the case where hiring costs depend only on vacancy posting. Indeed, the amplification in the response of labor market variables to technology shocks is very much reduced. To understand why the mechanism presented in Section 5.2 is weakened in the case of $\eta^q = 2$ consider the FOC for hiring,

FIGURE A2. Impulse responses to an expansionary monetary policy shock, vacancy costs only vs vacancy and hiring costs.



Note: Impulse responses to a 25 basis points monetary policy expansion shock obtained for two different parameterizations of η^q , both with "high" hiring costs $e = 5$. The orange (dashed) line uses the benchmark $\eta^q = 0.49$ implying the co-existence of both vacancy and hiring costs; and the purple (solid) line uses $\eta^q = 2$, implying vacancy costs only. All variables are expressed in percent deviations, except hiring, investment, and real rates, which are expressed in percentage points deviations. Output is specified net of hiring costs.

where now

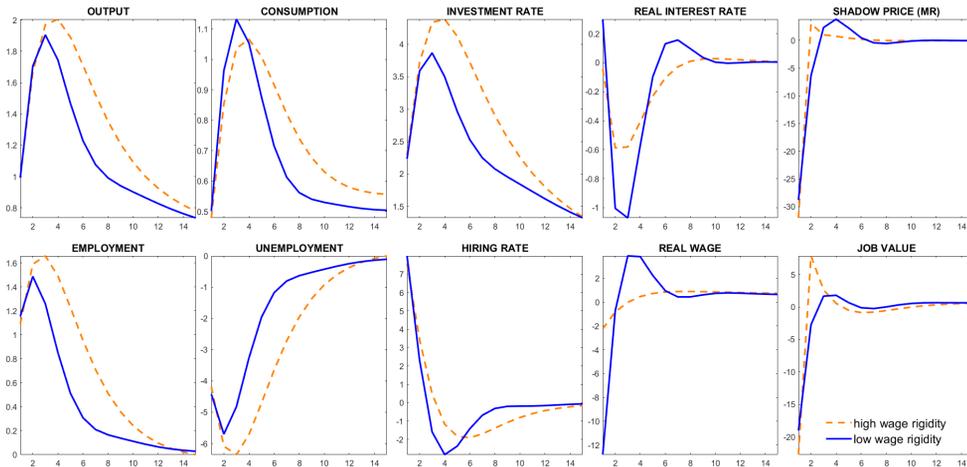
$$Q_t^N = \Psi_t g_{H,t} = \Psi_t e \frac{1}{q_t} \frac{V_t}{N_t} \frac{f(z_t, N_t, K_t)}{N_t},$$

As before, a fall in the shadow price Ψ_t engendered by an expansionary technology shock still decreases the marginal cost of hiring, thereby increasing vacancy creation. But the congestion externalities in the matching function imply a strong fall in the vacancy filling rate q_t , which in turn increases the marginal cost of hiring, offsetting the initial effect of Ψ_t . Note, that for values of η^q less than 2, as examined above, aggregate labor market conditions, expressed via q_t , matter less for the marginal cost of hiring, and the strong feedback effect of vacancy rates on the marginal cost of hiring is muted.

A.4 The role of wage rigidity

Figures A3 and A4 below compare impulse responses to technology and monetary policy shocks obtained under high wage rigidity (inertia parameter $\omega = 0.87$), and

FIGURE A3. Impulse responses to a positive technology shock: high vs. low Wage rigidity.



Note: Impulse responses to a 1 percent positive technology shock obtained for two different parameterizations of ω , both with "high" hiring costs $e = 5$. The orange (dashed) line uses the benchmark $\omega = 0.87$; the blue (solid) line uses $\omega = 0.10$, implying low wage inertia. All variables are expressed in percent deviations, except hiring, investment, and real rates, which are expressed in percentage points deviations. Output is specified net of hiring costs.

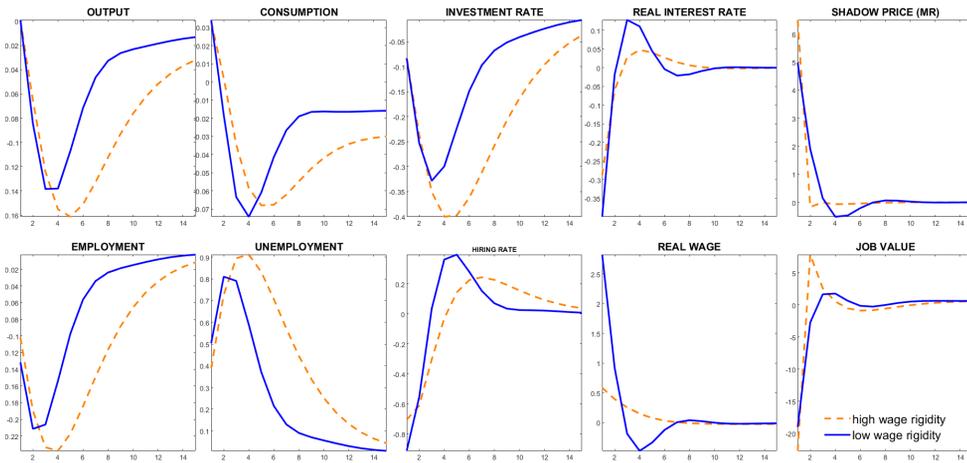
low wage rigidity ($\omega = 0.10$), assuming a high value of hiring frictions ($e = 5$), and setting all other parameter values as in Table A1.

A.5 Variations in the Taylor rule

It is well known that in NK models the dynamics of the endogenous variables are sensitive to the precise parameterization of the Taylor rule coefficients. For instance, a positive technology shock implies that the same level of demand can be achieved with less labor, so everything else equal the demand for labor falls. But at the same time inflation also drops, inducing a fall in the nominal interest rate via the Taylor rule, which in turn offsets the tendency for employment to decline. In equilibrium, employment can rise or fall, depending on the endogenous response of interest rates.

So, in order to show that the offsetting effect of hiring frictions on the standard NK propagation does not depend on the parameters of the Taylor rule, we have carried out the following robustness exercise. We take as a benchmark the version of the extended model parameterized with comparatively high frictions, i.e. $e = 5$. Under this parameterization an expansionary technology shock produces an increase in employment and an expansionary monetary policy shock produces a contraction

FIGURE A4. Impulse responses to an expansionary monetary policy shock, high vs. low wage rigidity.



Note: Impulse responses to a 25 basis points monetary policy expansion shock obtained for two different parameterizations of ω , both with "high" hiring costs $e = 5$. The orange (dashed) line uses the benchmark $\omega = 0.87$ implying high wage rigidity; and the blue (solid) line uses $\omega = 0.1$, implying low wage rigidity. All variables are expressed in percent deviations, except hiring, investment, and real rates, which are expressed in percentage points deviations. Output is specified net of hiring costs.

in output (Figures A-3 and A-5). To show that these substantial results are a genuine manifestation of the offsetting effect of friction costs, and not an artifact of a specific Taylor rule, we inspect impulse responses obtained by randomizing the Taylor rule coefficients over a broad parameter space, leaving all other parameters fixed at the values reported in Table A-1.

Specifically, we have generated 10,000 parameterization vectors, which differ only in the coefficients governing the Taylor rule. These parameter values are assigned by drawing randomly from uniform distributions defined over the support of $r_y \sim U(0,0.5)$, $r_\pi \sim U(1.1,3)$ and $\rho_r \sim U(0,0.8)$. Our results indicate that output responded negatively on the impact of a monetary stimulus in every single parameterization, and the sign of the response was never overturned one year or two years after the impact. Similarly, on the impact of the technology shock instead, employment responded positively in every single parameterization. The sign of the response was not overturned after one year in any of the parameterizations and remained in positive territory, after two years, in 99.8% of the parameterizations.

APPENDIX B: LOCAL PROJECTIONS ANALYSIS

In this Appendix we delineate the methodology and data series used and report variations on the LP analysis presented in the main text.

B.1 Methodology

We implement a local projections (LP) methodology to generate data-based IRFs. This methodology was suggested by Jordà (2005). Subsequently, several authors, including Jordà, Schularick, and Taylor (2020), have shown how to use this method with a LP-IV estimator, employing the shocks as instruments. Stock and Watson (2018) delineate and discuss the conditions of relevance and exogeneity under which external instrument methods produce valid inference on structural impulse response functions. The basic LP regression is the following:

$$s_{t+h} = c_h^s + \lambda_h^s \varepsilon_t^{TFP} + \Gamma_h^{s'} \mathbf{X}_t + e_{t+h}^s \quad (20)$$

Equation (20) can be understood as follows. On the LHS s_{t+h} is a predicted variable at horizon h . On the RHS the shock is denoted ε_t . Each regression has a constant (c_h^s) and an error term (e_{t+h}^s). \mathbf{X}_t is a vector of control variables. The estimated coefficients are λ_h^s and the vector Γ_h^s , respectively. A plot of the λ_h^s traces out the effect of the shock ε_t on the variable s_{t+h} , i.e., the impulse response function (IRF) of the variable to the shock. We compute Newey-West HAC standard errors.

The rationale for this methodology is that it is a direct forecasting method, as distinct from iterated forecasting, and puts fewer restrictions on the IRFs relative to VARs. Here we implement it for the TFP shock.

For the monetary policy shock we use the LP-IV method. The following equation is run at second stage:

$$s_{t+h} = c_h^s + \lambda_h^s \widehat{R}_t + \Gamma_h^{s'} \mathbf{X}_t + e_{t+h}^s \quad (21)$$

where the fitted interest rate \widehat{R}_t emerges from the first stage where one estimates:

$$R_t = a + bZ_t + \mathbf{c}'\mathbf{X}_t + v_t \quad (22)$$

In this equation a is a constant, R_t is the rate on the one year constant-maturity Treasury, Z_t is the instrument, which is the monetary policy shock ε_t^{MP} , there is

an error term v_t , and b and c are coefficients. Stock and Watson (2018) use this formulation to estimate the response of four key U.S. macroeconomic variables to a monetary policy shock ε_t^{MP} .

Estimation of the equations requires detrending non-stationary series and the choice of control variables (X_t). Detrending is done by (i) using a fourth-order polynomial trend function or (ii) working with log first differences. We compute Newey-West HAC standard errors.

B.2 Data for MP shocks

Table B1 presents the data series used. All of the variables used are in logs. When a variable x is a rate it is formulated as $\ln(1 + x) \simeq x$. The following table presents the details; sources are from FRED unless noted otherwise. Wherever relevant we converted monthly data to the quarterly frequency by averaging.

TABLE B1. Data used for MP shocks.

	symbol	definitions	source (FRED)
<i>real</i>	f_t	Real GDP (non-farm, business)	OUTNFB
	n_t	Employment rate (civilian)	CE16OV and CLF16OV
	u_t	Civilian Unemployment rate	UNRATE
<i>prices, inflation</i>	CPI_t	CPI	CPIAUCSL
	P_t^{COM}	Commodity (non-energy) Price Index	World Bank
<i>financial</i>	R_t	One-year constant maturity Treasury rate	GS1
	FFR_t	Effective Federal Funds rate	FEDFUNDS
	r_t	Real Interest rate; See note a	FEDFUNDS and CPIAUCSL
	EBP_t	Excess Bond Premium	Gilchrist and Zakrajšek (2012)
	MN factor 2 _t	Macro Factor 2; See note b	MN
<i>markup</i>	$lmu - cd$	Log of markup, CD, labor compensation	Nekarda and Ramey (2020)
<i>SW-factors</i>	4 factors	computed by Stock and Watson (2018)	Stock and Watson (2018)
<i>MP shocks</i>	ε_t^{MP}	Romer-Romer shocks; See note c	Romer and Romer (2004); Wieland and Yang (2017)

Note: a) We compute the real rate as the log of the ratio of the gross Federal Funds Rate rate to the gross CPI rate of inflation. b) We take factors from McCracken and Ng (2016), to be denoted MN. The interpretation of the MN Factor 2 is "term spreads, inventories." c) The Romer and Romer shock series is taken from Wieland and Yang (2017).

TABLE B2. Control variables in monetary policy shock projections.

1	$\varepsilon_{t-i}^{MP}, f_{t-j}, CPI_{t-j}, FFR_{t-j}$
2	controls in (1) + R_{t-j}, EBP_{t-j} , MN factor 2_{t-j} , mark-up $_{t-j}$, SW factors $_{t-j}$
3	controls in (1) + R_{t-j}, EBP_{t-j} , MN factor 2_{t-j} , mark-up $_{t-j}$
4	controls in (1) + SW factors $_{t-j}$
5	controls in (1) + R_{t-j}, EBP_{t-j}
6	controls in (1) + mark-up $_{t-j}, P_{t-j}^{COM}$
7	controls in (1) + R_{t-j}, EBP_{t-j} , MN factor 2_{t-j}
8	controls in (1) + MN factor 2_{t-j} , mark-up $_{t-j}, P_{t-j}^{COM}$

Note: Time index $j=1,2$

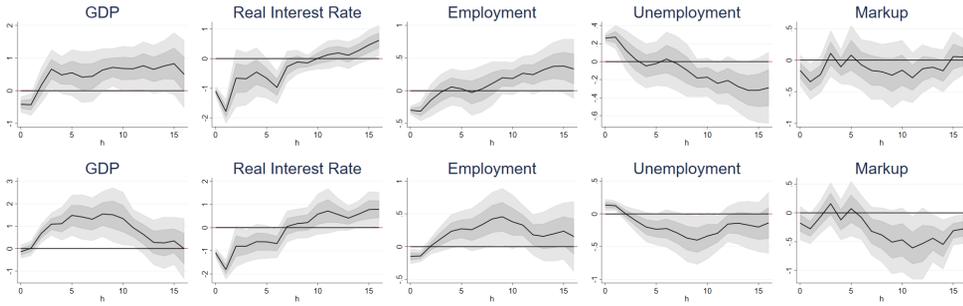
As to the 4 factors computed by Stock and Watson (2018), they state the following (page 942):

“including additional variables that are correlated with the shocks could further reduce the regression standard error, and thus result in smaller standard errors. One plausible set of such variables are principal components (factors) computed from a large set of macro variables. With this motivation, column (3) adds lags of four factors computed from the FRED-MD data set (McCracken and Ng, 2016).”

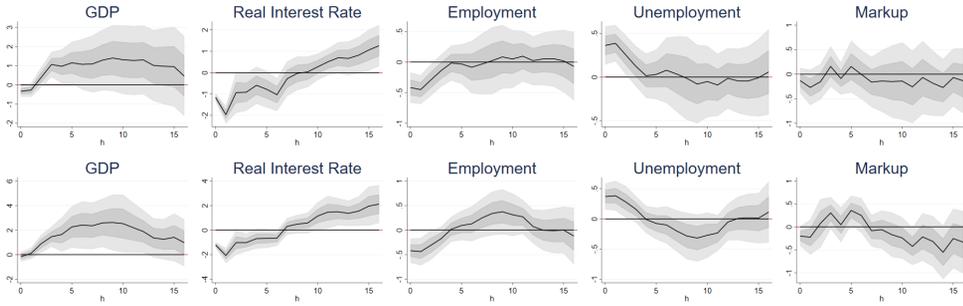
B.3 Results for MP shocks for alternative specifications

Table B2 and Figure B1 report alternative specifications for the control variables and the IRFs to an expansionary monetary policy shock. This is presented in the same format as Table 4 and Figure 6 in the main text.

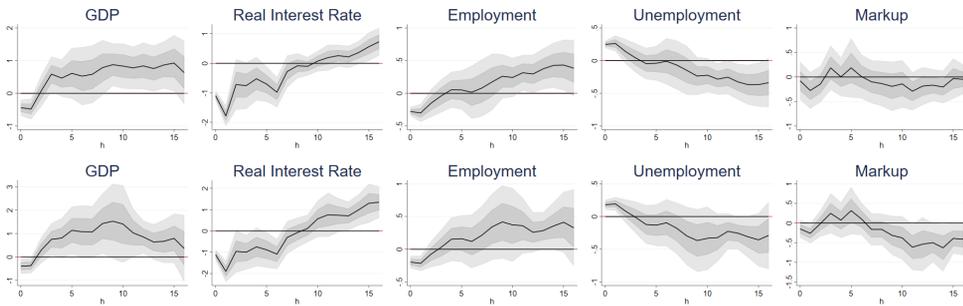
FIGURE B1. Expansionary monetary policy shock, LP-IV analysis



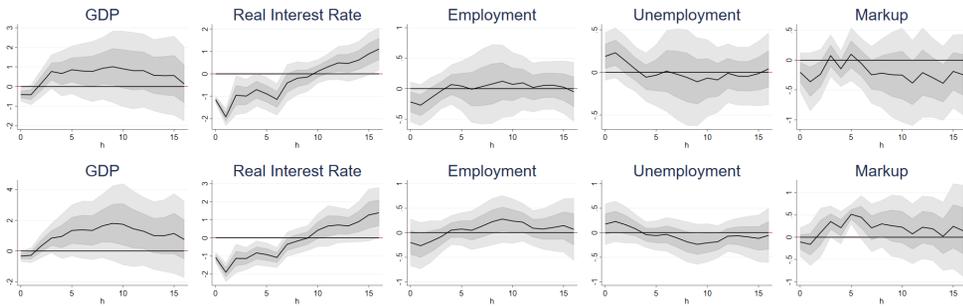
a. Fourth Order Time Trend Specifications, rows 3 and 4



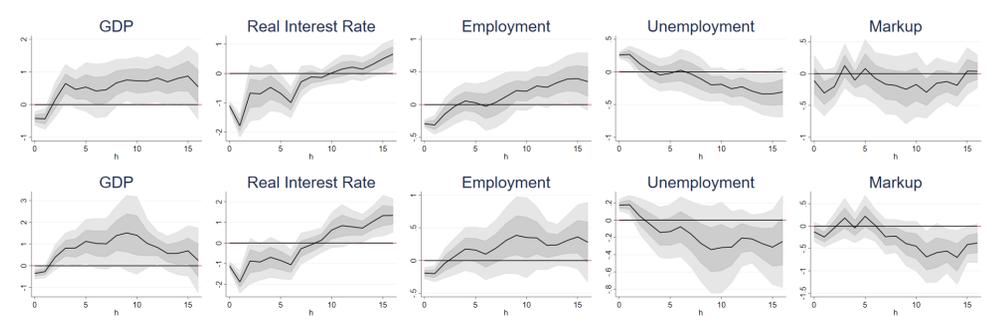
b. First Difference Specifications, rows 3 and 4



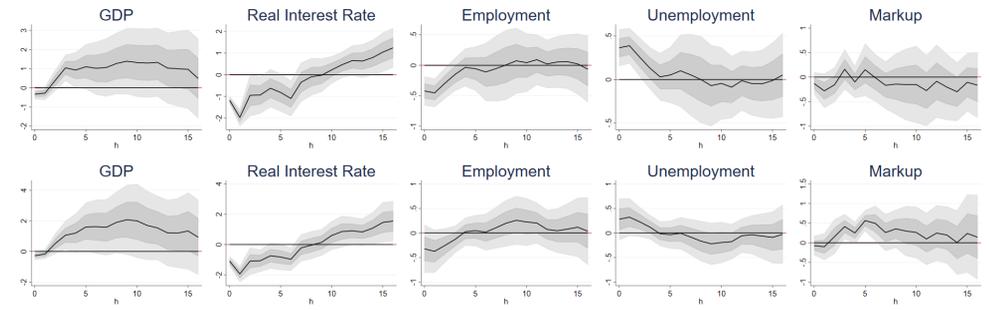
c. Fourth Order Time Trend Specifications, rows 5 and 6



d. First Difference Specifications, rows 5 and 6



e. Fourth Order Time Trend Specifications, rows 7 and 8



f. First Difference Specifications, rows 7 and 8

B.4 Data for TFP shocks

All of the variables used are in logs. When a variable x is a rate it is formulated as $\ln(1+x) \simeq x$. Wherever relevant we converted monthly data to the quarterly frequency by averaging. Table B3 presents the details.

TABLE B3. Data used for TFP shocks.

symbol	definitions	source (FRED)
f_t	Real GDP (non-farm, business)	OUTNFB
n_t	Employment rate (civilian, CPS)	CE16OV and CLF16OV
u_t	Unemployment rate	UNRATE
MN factor 1 _t	Macro Factor 1; See note a	MN
$R\&D$	Gross Domestic Product: Research and Development	Y694RC1Q027SBEA
$LFPR$	Civilian Labor Force Participation Rate	CIVPART
$55 + LFPR$	Civilian Labor Force Participation Rate: 55 year +	LNS11324230
$womenLFPR$	Civilian Labor Force Participation Rate: Women	LNS11300002
$lmu - cd$	Log of markup, CD, labor compensation	Estimates, Nekarda and Ramey (2020)
R_t	One-year constant maturity Treasury rate	GS1
r_t	Real Interest rate; See note b	FEDFUNDS and CPIAUCSL
ε_t^{TFP}	TFP Shock	Fernald (2014)

Note: a) We take factors from McCracken and Ng (2016), to be denoted MN. The interpretation of the MN Factor 1 is real activity b) We compute the real rate as the log of the ratio of the gross Federal Funds Rate rate to the gross CPI rate of inflation.

B.5 Results for TFP shocks: alternative specifications

Table B4 and Figure B2 report alternative specifications for the control variables and the IRFs to an expansionary TFP shock. This is presented in the same format as

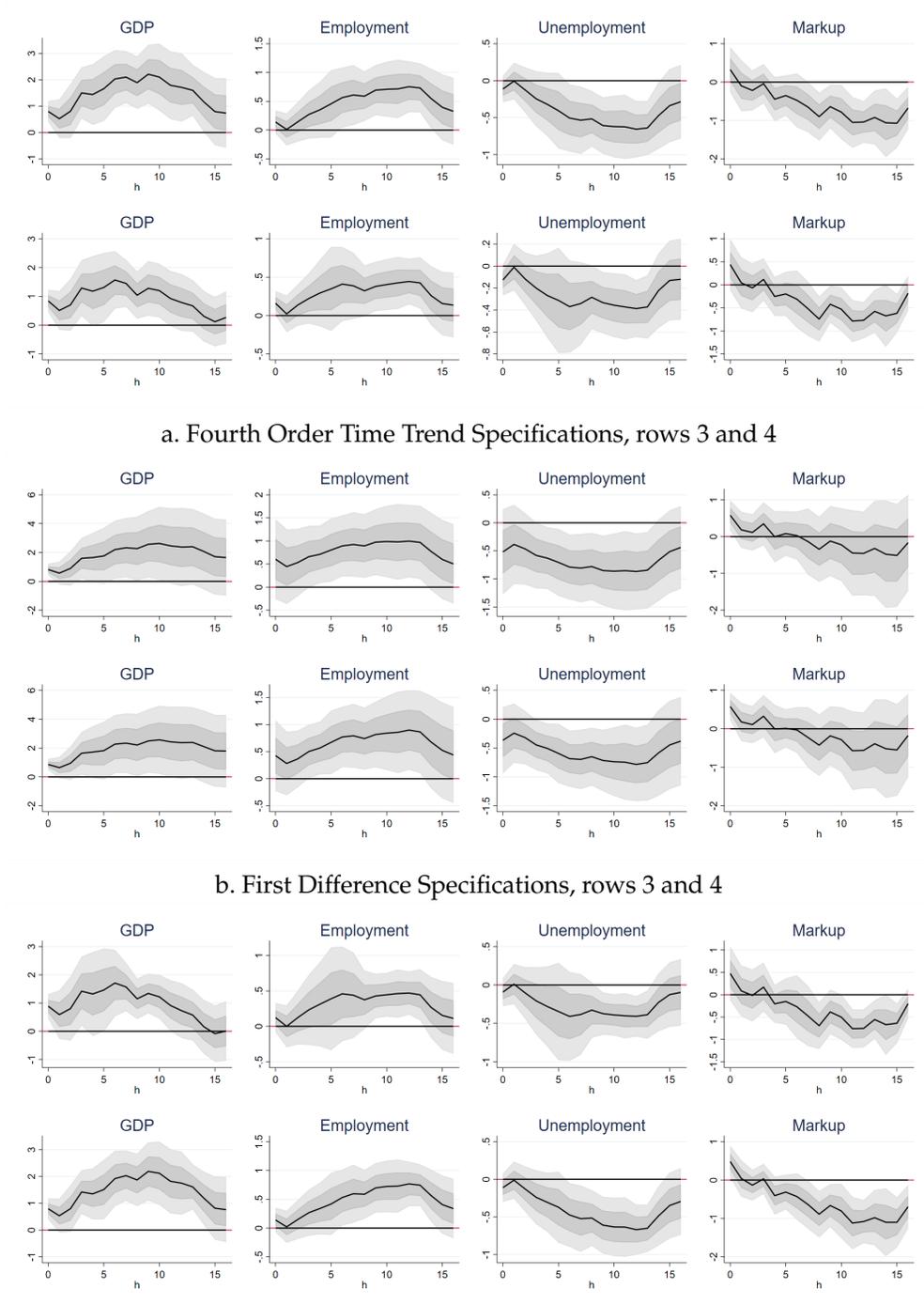
Table 5 and Figure 7 in the main text.

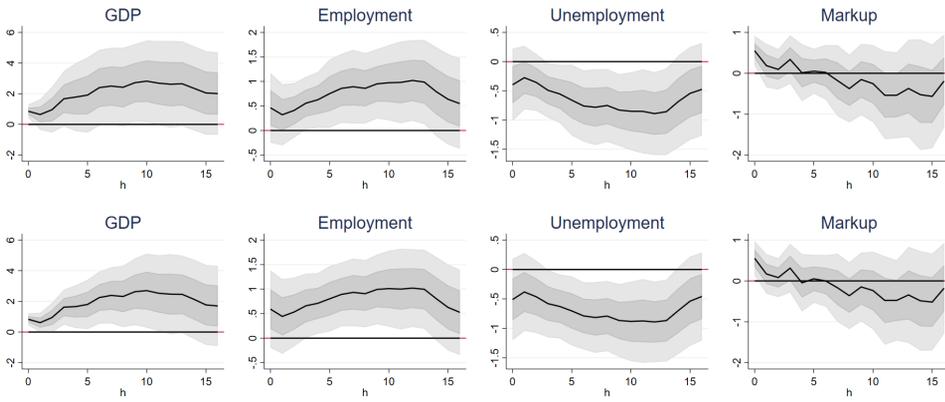
TABLE B4. Control variables in TFP shock projections.

1	$\varepsilon_{t-j}^{TFP}, f_{t-j}, MN \text{ factor } 1_{t-j}$
2	controls in 1 + mark-up _{t-j} , lnR&D _{t-j} , LFPR _{t-j} , femaleLFPR _{t-j} , 55+LFPR _{t-j} , R_{t-j}, r_{t-j}
3	controls in 1 + r_{t-j}
4	controls in 1 + lnR&D _{t-j} , LFPR _{t-j} , femaleLFPR _{t-j} , 55+LFPR _{t-j} , r_{t-j}
5	controls in 1 + LFPR _{t-j} , femaleLFPR _{t-j} , 55+LFPR _{t-j}
6	controls in 1 + mark-up _{t-j} , R_{t-j}, r_{t-j}
7	controls in 1 + lnR&D _{t-j} , R_{t-j}
8	controls in 1 + lnR&D _{t-j} , LFPR _{t-j} , femaleLFPR _{t-j} , 55+LFPR _{t-j}

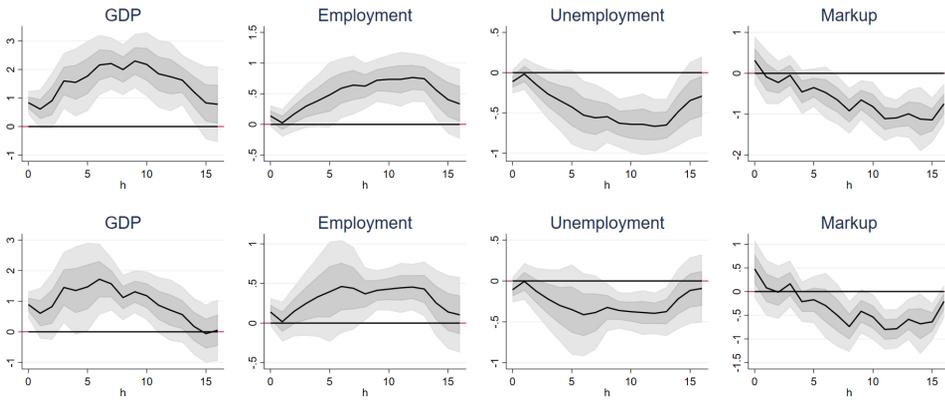
Note: Time index $j=1,2$

FIGURE B2. Expansionary technology shock, LP analysis

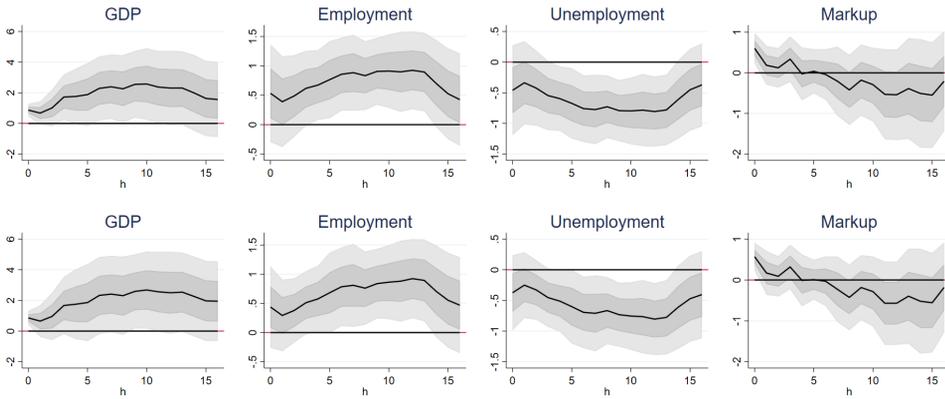




d. First Difference Specifications, rows 5 and 6



e. Fourth Order Time Trend Specifications, rows 7 and 8



f. First Difference Specifications, rows 7 and 8

APPENDIX: REFERENCES

- Christiano, L. J., M.S. Eichenbaum, and M. Trabandt (2016), "Unemployment and Business Cycles." *Econometrica*, 84, 4, 1523–1569.
- Den Haan W.J. , G. Ramey, and J.Watson (2000), "Job Destruction and Propagation of Shocks." *American Economic Review*, 90, 482–498.
- Galí, J. (2011), "Monetary Policy and Unemployment." Chapter 10 in B.M. Friedman and M.Woodford (eds.) *Handbook of Monetary Economics Vol. 3A*, 487-546, North Holland, Amsterdam.
- Gertler, M., L. Sala, and A. Trigari (2008), "An Estimated Monetary DSGE Model With Unemployment and Staggered Nominal Wage Bargaining." *Journal of Money, Credit and Banking*, 40, 1, 713-64.
- Hall, R. E. (2005), "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review*, 95(1), 50-65.
- Jordà, O. (2005), "Estimation and Inference of Impulse Responses by Local Projections." *American Economic Review*, 95, 1, 161-182.
- Jordà, O. M. Schularick, and A.M. Taylor (2020), "The Effects of Quasi-Random Monetary Experiments." *Journal of Monetary Economics*, 112, 22-40.
- McCracken, Michael W. and Serena Ng (2016), "FRED-MD: A Monthly Database for Macroeconomic Research." *Journal of Business & Economic Statistics*, 34:4, 574-589.
- Rotemberg, J. (1982), "Monopolistic Price Adjustment and Aggregate Output." *Review of Economic Studies*, 49, 517-31.
- Sala, L., U. Söderstrom, and A. Trigari (2013), "Structural and Cyclical Forces in the Labor Market during the Great Recession: Cross-Country Evidence" in F.Giavazzi and K.D. West (eds.). *NBER International Seminar on Macroeconomics 2012*, Chapter 8, 345- 404, University of Chicago Press.
- Silva, J., and M. Toledo (2009), "Labor Turnover Costs and the Cyclical Behavior of Vacancies and Unemployment." *Macroeconomic Dynamics*, 13 (S1), 76-96.
- Stock, J.H. and M.W. Watson (2018), Identification and Estimation of Dynamic Causal Effects in Macroeconomics Using External Instruments. " *The Economic Journal*, 128, 917–948.
- Wieland, J.F. and M.J. Yang (2017), "Financial Dampening." working paper.